Chapter 20

GROWTH MODELS WITH ENDOGENOUS POPULATION: A GENERAL FRAMEWORK

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1. Introduction

For most of human history, birth rates and death rates have fluctuated roughly in tandem so that population itself remained stable or grew only slowly. With the development of agriculture some 12 000 years ago, it is believed that both birth and death rates increased substantially, but that there was some acceleration in the rate of growth of population (Coale, 1974). A second major change occurred in Western Europe following what has been called "the second agricultural revolution" and preceding the Industrial Revolution (Bairoch, 1976). This was a remarkable fall in death rates followed only slowly by a fall in birth rates, so that population literally exploded. In the eighteenth and nineteenth centuries, the excess of population was relieved by the vast migrations of European population to the new lands of the Western Hemisphere and Oceania. It was against this backdrop that Malthus (1798, 1830) wrote.

Malthus' theory of population and growth is well-known: Passion between the sexes, unless checked by human misery, leads to a continual growth in population. Positive checks to population growth included "... war, disease, hunger, and whatever ... contributes to shorten the duration of human life". Preventative checks included abstinence from sexual relations, continence within marriage, and/or delay of marriage. But Malthus didn't think that even the preventative checks would operate to any great extent in the absence of the incentives forced on mankind by increasing misery. As long as living conditions did not deteriorate greatly, population would grow exponentially. Since, however, Malthus believed that food supplies and ultimately the means to human welfare more generally could only grow linearly, he predicted population growth with increasing immiserization until equilibrium was reached for a large population living under the most abysmal conditions.

That Malthus' dire prediction has not yet been realized is the result of many factors. First, in Western Europe and later in Eastern Europe, North America and Japan, as death rates, particularly infant and child mortality rates, fell, birth rates ultimately came down as well, although with a substantial lag. Second, agricultural productivity increased substantially and new lands were opened reducing population pressure in older settled areas and making more food and other resources available to support a growing world population. In the twentieth century, even as modern medical advance and public health investments have reduced death rates in other parts of the world, a similar pattern of falling birth rates, agricultural extension and intensification (recently through so-called "Green Revolution" technology), and general economic growth, has been followed in large parts of the world. (Africa is a notable exception.) The pattern of falling death rates followed after a lag by falling birth rates has been called the "demographic transition" (Beaver, 1975; Caldwell, 1982). But, to date, a rigorous theory about whether and how this demographic pattern might be linked to economic growth has proven elusive.

In this connection, it is important to make a distinction between endogenous population change and endogenous fertility. Models can be constructed in which there is a relation between economic and other factors and the size of, composition of, and changes in population, but in which no decision-making mechanism is presupposed. Purely biological models of animal populations in which food supplies or predator population limit the size of the population in question are of this character. The Malthusian theory, discussed above, comes close to this paradigm.

On the other hand, recent developments in population and family economics suggest many causal paths between the economic environment and *human* family formation and fertility decisions as well as the possibility that mortality may be influenced by families' decisions on the investments in human capital, in the form of health and nutrition, they make in their children and fertility decisions. In particular, recent *economic* theories of fertility focus on explicit family decision-making models in which optimal fertility choices are made in a utility-maximizing framework. Fertility is, of course, only one component of population change. In a closed population without migration, demographic composition and mortality also play a role over which families have little control.

The problem of explaining the demographic transition in these terms is to show how family decisions with respect to fertility, investment in the human capital of their children and their bequests to them in other forms of capital, and other variables interact over time to determine the size of the population and the stocks of capital, both human and physical, and the well-being of successive generations, and then to deduce the demographic transition as a possible outcome of these interactions.

In the early 1970s when the outlines of the "new home economics" were just emerging, Nerlove (1974, pp. S215–S217) speculated on what such a model might look like:

> Good nutrition and health care increase youngsters' chances of survival and may also affect their ability to absorb future investments in intellectual capital. To the extent that such investments increase the life span, particularly the span of years over which a person can be economically active, such an increase in quality will raise the return to investments in human capital which sons and daughters may later wish to make in themselves. To the extent that better health and nutrition result in a reduction in child mortality, they increase the satisfactions accruing to parents from other forms of investment which also raise child quality, for the returns to these investments may then be expected to be enjoyed over a longer period of time on average. Increases in longevity, particularly of an individual's economically productive years, increase the amount of human time available without increasing population; such an increase would tend by itself to lower the value of time per unit, but, as we know, most of the effects of better health care and nutrition occur in childhood and enhance the quality of a unit of time in later years more than increasing the number of children. On net balance, therefore, I would conjecture that better health and nutrition lower the costs of further investments in human capital relative to those in other forms of capital and increase the returns therefrom. ...

For reasons which I feel certain we do not fully understand, but which are due in part to the presence of children's utilities in the utility function of the family to which they belong, parents do desire to bequeath a stock of capital to their children. Since the stock of capital, material and intangible, human and nonhuman, is growing per capita in Western economies, one must assume that parents desire to pass along more than that which they received from their parents, or that institutions in the economy function in such a way as to induce this outcome. Irrespective of the motivation, however, the increasing value of human time must have an effect on the form in which this capital is passed on. As long as the rates of return to investments in human capital remain above, or fall more slowly than, the rates of return to investments in other forms of capital, parents will be induced to bequeath a greater part in the form of human capital. Thus the tendency toward increasing quality of children will be intensified by the bequest motive, despite the opposite tendency, resulting from the increasing cost of time, to invest in bequests which are less time-intensive. But as rates of return tend to equality over time – if they ever do – parents should tend to bequeath less in the form of human capital and more in the form of financial and physical capital. [In equilibrium, rates of return will be the same. If they differ, parents will invest in those assets yielding the highest rates of return.] Nonetheless, as long as investment in human capital occurs, the value of a unit of human time will continue to rise with increases in the stock of capital per capita, reinforcing the tendency to fewer children of ever-higher quality. Substitution will occur in favor of fewer children of higher quality and perhaps eventually against both quality and quantity of children in favor of commodities and knowledge. [There is considerable evidence that an increasing proportion of total capital formation in this century has occurred in the form of human capital (Schultz, 1961, 1971, 1973) which suggests] ... that we may be far from the point at which such substitution begins to take place against children, quality and quantity combined.

The outlines of a revised Malthusian model begin to emerge, albeit dimly, from the foregoing conjectures and speculations. In this model, the value of human time and changes in that value over time are pivotal, and the limitations imposed by natural resources are mitigated, if not eliminated, by technological progress and increases in the stock of knowledge and of capital, both human and nonhuman. The main link between household and economy is the value of human time; the increased value of human time results in fewer children per household, with each child embodying greater investments in human capital which in turn result in lower mortality and greater productivity of the economically active years. Such greater productivity in turn further raises both the value of a unit of time and income in the subsequent generation and enables persons of that generation to make efficient use of new knowledge and new physical capital. Eventually, rates of return to investments in physical capital, new knowledge, and human capital may begin to equalize, but as long as investment occurs which increases the amount of human capital per individual, the value of a unit of human time must continue to increase. It is not possible to say whether the diminishing ability of a human being to absorb such investment would eventually stabilize the number of children per household and at what level, given the satisfactions parents obtain from numbers of children as well as their quality. Nonetheless, over time the model does predict in rough qualitative fashion declining rates of population growth (perhaps eventually zero rates or even negative rates for a time) and declining rates of infant mortality. These are the main features of the demographic transition.

What may have happened sometime in the nineteenth and early twentieth centuries in the West was that a small exogenous shock which reduced infant and child mortality set off a cumulative process of investment in better health and nutrition and in public health leading to a surge in economic growth and population but eventually resulting in substitution of quality in the form of further human capital investments for numbers of children. And what may be happening in many places in the world today is the same cumulative process now set off by the import of modern medical knowledge and public health technology. But the occurrence of the demographic transition in these areas depends, if these conjectures are valid, on the existence of opportunities for, and absence of obstacles to, further investments in human capital.

The World Bank (1992: p. 26) projects that between now and 2160 the current world population of 5.5 billion will about double or more than quadruple depending on the rapidity of the demographic transition in those countries of the world which have not yet experienced it or where it is not fully complete. Understanding how and why the transition occurs is thus a matter of great importance if we, of the present generation, are to formulate appropriate economic and demographic policies, for such policies will determine whether world population stabilizes at moderate levels and a relatively high standard of living, or at high levels with a poor quality of life for the majority. This Chapter seeks to provide a framework for further analysis. Limitations of the present state of knowledge more than limitations of space preclude any definitive models which show the possibility of such transitions and reveal the circumstances under which they may occur.

The elements of a complete theory of the relation between population and economic growth along the lines envisaged would include a theory of family decisions with respect to fertility, investments in the human capital of their children, and bequests to them, in response to their expectations of future rates of return, income and prices, and embed these in a dynamic general equilibrium model, which would determine these rates as functions of state variables, such as population and the stocks of human and physical capital. Mortality, especially infant and child mortality, would not be wholly exogenous but would depend, in part, at least at very low levels of income, on the investments in the health and nutrition that parents were prepared to make in their children at the expense of their own consumption. We do not carry out this plan in this Chapter; a full development is left for others who may be so inspired, or for our own subsequent research. What we do attempt is to lay out a general framework into which the elements of such an economic theory of the demographic transition can be fit, to survey briefly recent related work in the "new home economics" and growth literatures, and assess the linkages among stocks, flows, and rates of return in this context.

We begin with a formal analysis of models of economic growth in which population is endogenous in the sense that its rate of change over time depends on per capita consumption or wages without explicit determination of fertility within a utility-maximizing model of family decision-making. We develop a general framework for the analysis of economic growth with endogenous population and three factors of production, physical capital, labor, and a third, unspecified factor Z. The factor may be a fixed or a renewable natural resource or a stock of knowledge, which contributes generally to the production of consumption goods and additional physical capital but which is not subject to control by individual economic agents.

We adopt a discrete time formulation in order to facilitate intergenerational analysis and the integration of models of endogenous fertility. We show that if production technology is homogeneous of degree one in the three factors and if the dynamic equation characterizing the law of motion of the third factor Z is also homogeneous of degree one in all three factors (i.e., if Z were truly variable), then the economic growth with endogenous population may be modeled as a dynamic planar (two-dimensional) system. The analysis of global and local properties of such a planar system may be carried out using methods developed in Nerlove (1993).

We begin with the Solow–Swan model (Solow, 1956; Swan, 1956), both of whom suggested (Solow: p. 91 and Swan: p. 339) that their model might be modified by introducing a simple form of endogenous population by assuming that the rate of growth of population depends on the real wage or per capita consumption. Growth with exogenously growing population is a very special case of the general two-dimensional system developed here. In fact, this case and the case in which the rate of growth depends on per capita consumption is obtained by eliminating the factor Z and thus reducing the system to one-dimensional dynamics. Niehans' (1963) model in which both savings and population are endogenously determined at the aggregate level in a neoclassical, constant-returns-to-scale context is a special case which is also one-dimensional. Finally, we provide a detailed analysis of the full model in which both population and savings are endogenous. We also consider a model in which there is a third factor of production, which is not under the direct control of economic agents but which nonetheless affects the output obtainable from capital and labor. This model is inspired by recent work of Lee (1986) in which he tries to encompass the theories of Malthus and Boserup in a single model. Our third factor may be environmental or other natural resources or a stock of technology and is unpriced. See, for example, Nerlove (1991) and Raut and Srinivasan (1993). The owners of capital are assumed to receive the surplus, so that total product can be exhausted despite the unpriced nature of the third factor. The full power of the planar (two-dimensional) framework is exploited in this context.

Recent work on endogenous growth focuses on human capital and its joint determination at the family level with fertility decisions and the effects of human capital investments on mortality. The important distinction between human capital and physical capital or natural resources or general knowledge is that human capital is fully embodied in the human agent and therefore affects production only through the individual and is extinguished with the death of the individual. This fact makes it possible to continue the analysis largely within a planar context within the general framework outlined here, although such considerations underscore the need for a deeper analysis of family decisions, the elements of which we next sketch.

We close Section 2 with a brief review of the new directions in growth theory initiated by Lucas (1988) and Romer (1986). This theory emphasizes increasing returns and the effects of a growing stock of knowledge. It is designed to eliminate exogenous technical change as the main source of growth and to explain the continuing divergence in rates of per capita income growth, in contrast to the prediction of neoclassical growth theory that growth rates should converge, and is of limited usefulness in answering the central question addressed here.

Next we recall the basic theory of household choice with respect to consumption, saving, fertility and investment in the health and future welfare of their offspring. Our discussion suggests how the development of Chapter 5, pp. 53–58, of Nerlove et al. (1987) may be extended to models of utility-maximizing behavior which encompass decisions not only on how many children to have but how much to invest in their future well-being and in physical or financial resources available to parents in future time periods and, subsequently, to their children. We are particularly concerned with the effects of infant and child mortality and the ability of parents to influence these risks by devoting additional resources to the care and nutrition of their children. The purpose of this discussion is to suggest what further research might be necessary to provide an underpinning for an economic theory of the demographic transition advanced above.

The chapter concludes with a brief review of the nature of the interactions between household decisions and the main stock and flow variables which characterize the evolution of the economy over time: household decisions with respect to fertility and the investments in human capital made in their children and bequests to them, on the one hand; and population, the stock of human capital embodied in that population, and the stock of physical capital, on the other. Given the rules of distribution and tax–subsidy policies, the latter, state, variables, determine, via production technology, per household incomes and the rates of return to human and physical capital; these, in turn, are the main variables to which households respond in addition to the rates of mortality, particularly infant and child mortality. At low levels of income and of human capital in the form of investments in health and nutrition, these rates themselves may be partly endogenous. We then turn to a discussion of the effects of motives for intergenerational transfers such as old-age security and bequests, and introduce two-sided altruism as way of endogenizing such transfers. The possibility of strategic behavior and its effects on capital accumulation and population growth is briefly touched upon (see Cigno, 1991, Chap. 9; Raut, 1993). In the model of Azariadis and Drazen (1993) such strategic considerations play a pivotal role in explaining overall population growth and its sectoral composition.

2. Models of economic growth with endogenous population

2.1. Solow–Swan

Consider first the standard Solow–Swan model with exogenous population growth in discrete time form: Let Y_t = output, K_t = capital stock, N_t = labor force assumed to be the same as population, S_t = savings, I_t = investment, s = the savings rate, δ = depreciation rate, $\bar{n}$ = the exogenous rate of growth of population and labor force. Production can be represented by a constant returns to scale function:

$$Y_t = F(K_t, N_t) \qquad \text{or} \qquad y_t = f(k_t), \tag{2.1}$$

where $y_t = Y_t/N_t$, $k_t = K_t/N_t$, and $f(k) = F(k, 1)$. Solow–Swan assume that savings equals gross investment and is a constant fraction s of output:

$$I_t = S_t = sY_t. \tag{2.2}$$

The change in the capital stock equals gross investment minus depreciation:

$$K_{t+1} = (1-\delta)K_t + I_t = sF(K_t, N_t) + (1-\delta)K_t. \tag{2.3}$$

Population grows exogenously at a rate $\bar{n}$:

$$N_{t+1} = (1+\bar{n})N_t. \tag{2.4}$$

Thus

$$k_{t+1} = \frac{sf(k_t)+(1-\delta)k_t}{1+\bar{n}} = g(k_t), \quad k_0 \text{ given}. \tag{2.5}$$

The dynamics of the Solow–Swan model are entirely described by the path of k_t, the capital–labor ratio, since population grows exogenously, capital depreciates at a fixed rate, and gross investment is proportional to output.

The existence of stationary solutions to Eq. (2.5), i.e. k^* for which

$$k^* = g(k^*) \tag{2.6}$$

and the local stability of such solutions depend on the shape of the function g. The conditions which yield a nonnegative globally stable steady-state solution are the following:

$$g'(0) > 1,$$

$$g'(k) < 1, \quad \text{for some } k > 0,$$

and g is concave. These properties follow if the production function satisfies:

$$f(0) = 0,$$

$$f'(0) > \frac{\delta + \overline{n}}{s},$$

$$f'(k) < \frac{\delta + \overline{n}}{s}, \quad \text{for some } k > 0,$$

and f is concave. A stationary solution k^* is locally stable if $|g'(k^*)| < 1$. Clearly $k^* = 0$ is unstable. Under concavity of f, whenever Eq. (2.6) holds for some $k^* > 0$, then there can be no other $k^* > 0$ for which Eq. (2.6) holds and at that point $|g'(k^*)| < 1$, so the solution is necessarily unique.

In the model described above, substitute an equation determining the growth rate of population endogenously. Continue to assume that the savings rate is exogenously fixed. Suppose simply that the rate of growth of population depends on the level of per capita consumption:

$$\frac{N_{t+1}}{N_t} = 1 + n[(1-s)f(k_t)] = h(k_t), \quad h' > 0 \tag{2.7}$$

and $n(c_{\text{m}}) = 0$ for some level of per capita consumption, $c_{\text{m}} = f(k_{\text{m}})$. Then, in place of Eq. (2.5), the function $g(k_t)$ is now defined as

$$k_{t+1} = \frac{sf(k_t) + (1-\delta)k_t}{h(k_t)} = g(k_t), \quad k_0 \text{ given}. \tag{2.8}$$

The capital–labor ratio continues to determine the dynamics of the economy, that is, the system remains univariate, but is now more complex since h in the denominator of g now depends on k_t.

In this case, however, concavity of f and conditions on s and δ no longer guarantee the existence of stationary points nor do they determine unambiguously the local stability or instability of such equilibria. However, considerable insight into the location and properties of nontrivial steady states can be obtained by comparing them with the steady states of the Solow–Swan model with *exogenous* population growth. Let $\bar{k}^*$ be the stationary point of the system (2.5). Then,

$$\bar{k}^* = \frac{sf(\bar{k}^*) + (1-\delta)\bar{k}^*}{1+\bar{n}}$$

or

$$\frac{\bar{n}+\delta}{s}\bar{k}^* = f(\bar{k}^*). \tag{2.9}$$

Provided the conditions on the production function previously specified are satisfied, $\bar{k}^* = 0$ is a stationary point and $f(k)$ intersects a straight line through the origin with slope at a point $(\bar{n}+\delta)/s$ at a point $\bar{k}^* > 0$ as well.

Let us use the same notation to denote $n(k) = n[(1-s)f(k)]$. When population is endogenous, from Eq. (2.8), we have

$$\left[\frac{n(k^*)+\delta}{s}\right]k^* = f(k^*), \tag{2.10}$$

where, k^* denotes the stationary solution of Eq. (2.8). Comparing Eqs. (2.9) and (2.10), we note that while the left-hand side of Eq. (2.9) which corresponds to Solow–Swan model with exogenous population growth is a linear function of k, in the case of endogenous population, Eq. (2.10), it is a nonlinear function $\rho(k)$ given by

$$\left[\frac{n(k)+\delta}{s}\right]k = \rho(k).$$

As before, a stationary point is characterized by $\rho(k^*) = f(k^*)$. The properties of $\rho(k)$ depend on the function $n(k)$. If $n(k)k \to 0$ as $k \to 0$, and therefore y and $(1-s)y \to 0$, $\rho(0) = 0$. Let us assume that $n(k)$ is increasing in k, positive for k greater than some small value and $n(k) > \bar{n}$ for some $k > 0$. Since

$$\rho'(k) = \frac{n(k)+\delta}{s} + \frac{n'(k)\cdot k}{s},$$

which must be greater than $(\bar{n}+\delta)/s$ as $n' > 0$, there exists a unique k_0 such that $n(k_0) = \bar{n}$. At this point $\rho(k)$ crosses the line

$$\left[\frac{\bar{n}+\delta}{s}\right]k$$

and, under the assumptions made, lies everywhere above it. If the form of $\rho(k)$ is such that $k_0 < \bar{k}^*$ then the stationary capital–labor ratio, k^*, of the Solow–Swan model with endogenous population is less than $\bar{k}^*$, the capital–labor ratio of the Solow–Swan model with exogenous population; otherwise, $k^* > \bar{k}^*$. The first case is shown in Fig. 1.

In general, however, $n(k)$ may be increasing for smaller values of k and eventually turn down and recross the line $\bar{n}$ with $n' < 0$. Then another equilibrium may occur at a capital–labor ratio greater than $\bar{k}^*$. Alternatively, $n(k)$ may fall after very low levels of the capital–labor ratio are reached and may never reach the level $\bar{n}$. In this case there may be no nontrivial stationary point or an equilibrium only at a very large value of the capital–labor ratio. It is clear that merely endogenizing population growth at the macro level does not shed light on the shape of $n(k)$ and thus on the nature of dynamics; a utility-maximizing model should be used to elucidate the nature of the function $n(k)$, as we attempt in Section 3.4.

Suppose we have found a nontrivial steady-state solution to the Solow–Swan model with endogenous population growth. What are its dynamic properties? Differentiating g with respect to k in Eq. (2.8) and utilizing Eq. (2.10), we have

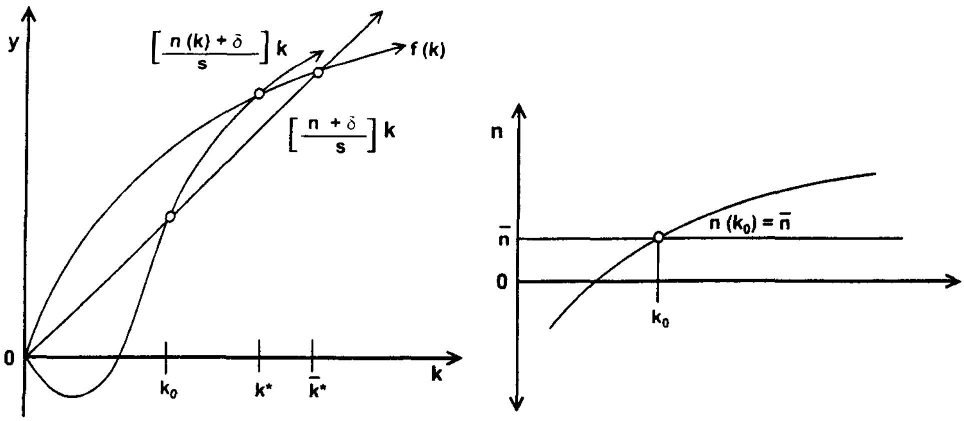

Fig. 1.

$$g'(k^*) = \frac{sf'(k^*) + (1-\delta) - k^*n'(k^*)}{1+n(k^*)}. \tag{2.11}$$

This expression is greater than −1 if

$$f'(k^*) > \frac{-[1+n(k^*)] - (1-\delta) + k^*n'(k^*)}{s},$$

which is satisfied unless $n'(k^*)$ is very large and positive. Let us assume that $n(k)$ is such that the above is satisfied. For local stability analysis, it suffices to focus on whether $g'(k^*) \gtreqless 1$. This will be the case according as

$$f'(k^*) \gtreqless \frac{n(k^*) + \delta + k^*n'(k^*)}{s} = \rho'(k^*). \tag{2.12}$$

It follows that, if $\rho(k)$ crosses $f(k)$ from below in Fig. 1, the stationary point so determined is stable, and, if from above, the stationary point is unstable.

As shown in Fig. 1, $\rho(k)$ is essentially a transformation of $n(k)$, so everything depends on the behavior of this function which may be quite nonmonotonic in $(1-s)y$ and therefore in k. In contrast to the usual Solow–Swan model with exogenous population growth and the usual assumptions about the production function, endogenous population entails the possibility of multiple equilibria and instability of some of the equilibria. Another interesting point is that even if the rate of population growth at a stationary point is the same as the corresponding rate which would have led to that stationary value when population is assumed exogenous, i.e., even if $n(k^*) = \bar{n}$, it does not follow that the equilibrium is stable, in contrast to the Solow–Swan model with exogenous population. From Eq. (2.12), the condition is

$$f'(k^*) \gtreqless \frac{\bar{n} + \delta}{s} + \frac{k^*n'(k^*)}{s},$$

so that even if the equilibrium would have been stable with exogenous population, i.e.,

$$f'(k^*) < \frac{\bar{n} + \delta}{s},$$

we may nonetheless have

$$f'(k^*) > \frac{\bar{n} + \delta}{s} + \frac{k^*n'(k^*)}{s}$$

if $n'(k^*) < 0$. In Section 3.4, however, we show that utility maximization implies $n'(k) > 0$ and thus demonstrate how a utility-maximizing model of endogenous fertility can clarify the dynamic properties of the growth model.

Example

To illustrate, suppose that population growth is simply equal to the ratio of actual consumption to some minimal, positive, level of consumption per capita, c_m:

$$h(k_t) = \frac{(1-s)f(k_t)}{c_m}. \tag{2.7'}$$

This formulation is more general than it may seem since it can be derived by appropriate choice of the units of output and consumption per capita from a linear approximation to the function $n(\ )$ in Eq. (2.7):

$$h(k) = 1 + \gamma[(1-s)f(k) - c_m].$$

If we choose $\gamma = 1/c_m$, then $h(k)$ is equal to $(1-s)f(k)/c_m$.

At a stationary point k^*, we can write

$$g'(k^*) = \frac{sf(k^*) + (1-\delta)}{h(k^*)} - e^*, \tag{2.11'}$$

where

$$e^* = \frac{h'(k^*)k^*}{h(k^*)}$$

is the elasticity of the rate of population growth factor at the stationary value of the capital–labor ratio; in this case

$$e^* = \frac{f'(k^*)}{f(k^*)}k^*$$

depends only on the production function. For a Cobb–Douglas production function,

$$f(k) = k^\sigma, \quad 0 < \sigma < 1, \tag{2.1'}$$

so that $e^* = \sigma$. In general, of course, e^* depends on the response of fertility and mortality to increases in the capital–labor ratio and, therefore, in income. In the case of a

Cobb–Douglas production function, $1 > e^* > 0$, but for more general models e^* may be negative.

The point $k = k^*$ is locally stable if $|g'| < 1$. From Eq. (11′), local stability implies

$$\frac{-(1-e^*)(1+n^*)-(1-\delta)}{s} < f'(k^*) < \frac{(1+e^*)(1+n^*)-(1-\delta)}{s},$$

where n^* is the rate of growth of population at that point. As we saw, it is not possible to say whether this condition is satisfied in general. However, for a Cobb–Douglas function, Eq. (1′), the problem is somewhat simpler, since $e^* = \sigma$ and $1 + n^* = (1 - s)y^*/c_{\mathrm{m}}$, where y^* is per capita output at the stationary point. Then

$$g'(k^*) = \frac{s\sigma y^*/k^* + (1-\delta)}{(1-s)y^*/c_{\mathrm{m}}} - \sigma \lesseqgtr 1 \tag{2.11″}$$

according as

$$s\sigma + (1-\delta)\frac{k^*}{y^*} \lesseqgtr (1+\sigma)(1-s)\frac{k^*}{c_{\mathrm{m}}}.$$

Unless c_{m} is very large relative to k^*, this condition will generally be satisfied as < for $0 < s, \sigma, \delta < 1$. Thus, there exists a locally stable unique nonzero steady state for the extended Solow–Swan model in this example.

2.2. Niehans

Niehans (1963) develops a model in which both savings and population growth vary endogenously. The basic structure of Niehans' model turns out to be very similar to the Solow–Swan model with endogenous population growth, so our analysis can be brief. Population and labor force are again equated, but now there exists a "capitalist" class, whose numbers do not matter, and who save according to the excess or shortfall of the return per unit of capital from some fixed rate. Population, on the other hand, grows or declines according to the excess or shortfall of the wage from some minimum. Let w_t = per capita wage of labor and r_t = return per unit of capital. Assuming the same constant-returns-to-scale production function as before, Eq. (2.1), and that capital and labor are each paid their marginal products,

$$r_t = f'(k_t) > 0, \tag{2.13}$$

$$w_t = f(k_t) - k_t f'(k_t) > 0. \tag{2.14}$$

Then, in place of Eqs. (2.2) and (2.4) we have

$$I_t = s(r_t)Y_t, \quad s' > 0, \quad s(0) = 0, \tag{2.15}$$

and

$$N_{t+1} = [1 + n(w_t)]N_t, \quad n' > 0, \tag{2.16}$$

and $n(w) < 0$ for w less than some minimum wage.

It is easy to see that the Niehans model is basically a minor modification of the Solow–Swan model in which the growth of the economy is entirely determined by the dynamics of the capital–labor ratio and population according to

$$\frac{N_{t+1}}{N_t} = h(k_t) = 1 + n[f(k_t) - k_t f'(k_t)] = 1 + n(k_t). \tag{2.16'}$$

Substitution of Eqs. (2.13) and (2.15) in Eq. (2.3) yields

$$k_{t+1} = \frac{(1-\delta)k_t + s[f'(k_t)]f(k_t)}{1 + n(k_t)} = g(k_t). \tag{2.17}$$

Eqs. (2.16′) and (2.17) are a model identical to Solow–Swan except that the savings rate now depends on the capital–labor ratio via the marginal product of capital. Once again, using the same notation $s(k) \equiv s(f'(k))$, all the conditions deduced above can be repeated with the following simple modification:

$$\rho(k) = \frac{n(k) + \delta}{s(k)}.$$

$\rho(k)$ is, however, now not a simple linear transformation of $n(k)$ as it was when only population was assumed to be endogenous and the savings rate exogenously determined. The following example shows that a nontrivial stationary point may not exist.

Example

Suppose

$$f(k) = k^\sigma, \quad 0 < \sigma < 1,$$

and the rate of population growth and of savings are proportional to some minimum wage, w_m, and minimum rate of return, r_m, respectively:

$$n(k_t) = \frac{f(k_t) - k_t f'(k_t)}{w_{\mathrm{m}}},$$

$$s(k_t) = \frac{f'(k_t)}{r_{\mathrm{m}}}.$$

Substitution then yields

$$\rho(k) = \left(\frac{(1-\sigma)y + \delta w_{\mathrm{m}}}{\sigma y}\right)\left(\frac{k r_{\mathrm{m}}}{w_{\mathrm{m}}}\right),$$

where $y = f(k) = k^{\sigma}$. For k^* to be stationary, $\rho(k^*) = f(k^*)$ as before so

$$\frac{w_{\mathrm{m}}}{r_{\mathrm{m}}}\sigma(k^*)^{2\sigma-1} - (1-\sigma)(k^*)^{\sigma} = \delta w_{\mathrm{m}}\,.$$

If $0 < \sigma \leq 1/2$, the left-hand side of this expression is a decreasing function of k^*. Moreover, it is zero when $k^* = 0$. Therefore no nontrivial stationary point exists.

2.3. *Malthus–Boserup*

Models such as Solow–Swan or Niehans with endogenous population growth and a constant-returns-to-scale, two-factor production function yield univariate dynamics in the capital–labor ratio. The major difference is that the dynamic behavior becomes more complex with endogenous population growth since it no longer depends solely on concavity properties of the production function and the values of a few exogenously determined parameters. In this subsection, we develop a model based on a three-factor production function inspired by recent work of Lee (1986) on Malthus and Boserup. In our model, labor receives its marginal product but the rest, the "surplus", goes to capitalists who save all of it. Because a third factor of production, which may be the stock of knowledge or a fixed resource such as land or a renewable resource such as environmental quality, is involved, this model also encompasses nonconstant returns to scale in the two-factor case with labor and capital. The model generally requires two dimensions to describe its dynamic behavior. It cannot be reduced to univariate dynamics but requires planar analysis, as discussed by Nerlove (1993).

Let Z be a variable which denotes the stock of technological knowledge, of environmental quality or other renewable resources, or of a fixed resource such as land or one which may be permanently depleted through use. In general, we will assume that Z varies over time. The case when Z is fixed is then an important special case, the one which Malthus presumably had in mind. Generally, however, Z may vary over time, reversibly or irreversibly, in response to levels or changes in the stock of physical capital or population. Boserup's arguments suggest a reversible process in response to population pressure.

To describe production, we replace Eq. (2.1) by

$$Y_t = F(K_t, N_t, Z_t),$$

which we assume to be constant returns to scale in all three factors. It will be convenient to express everything in per capita terms for which we use lower case letters. Thus

$$y_t = f(k_t, z_t) = F\left(\frac{K_t}{N_t}, 1, \frac{Z_t}{N_t}\right) \tag{2.18}$$

and $F_N = f - kf_k - zf_z$ is the marginal product of labor. Thus, the condition that labor is paid its marginal product becomes

$$w_t = f(k_t, z_t) - k_t f_k - z_t f_z.$$

Assume that labor saves nothing and that the growth of population (labor force) is determined by w_t, which is thus per capita consumption:

$$\frac{N_{t+1}}{N_t} = 1 + n[f(k_t, z_t) - k_t f_k(k_t, z_t) - z_t f_z(k_t, z_t)] = 1 + n(k_t, z_t). \tag{2.19}$$

If the entire surplus is saved and can be used only to augment the capital stock, Eq. (2.2) is replaced by

$$\begin{aligned} I_t &= K_t F_K + Z_t F_Z \\ &= Y_t - N_t w_t \\ &= N_t(y_t - w_t). \end{aligned}$$

Thus,

$$k_{t+1} = \frac{(1-\delta)k_t + (y_t - w_t)}{1 + n(k_t, z_t)} = g(k_t, z_t). \tag{2.20}$$

The function g depends only on k_t and z_t since $y_t = f(k_t, z_t)$ and w_t is also a function of k_t and z_t.

We assume that the evolution of Z is governed by

$Z_{t+1} = H(K_t, N_t, Z_t)$,
where H is homogeneous of degree 1 so that we can write

$$z_{t+1} = \frac{\psi(k_t, z_t)}{1+n(k_t, z_t)} = h(k_t, z_t), \tag{2.21}$$

where $\Psi(k_t, z_t) = H(k_t, 1, z_t)$. The system (2.20) and (2.21) is a planar system in k_t and z_t, of the kind described in Nerlove (1993).

Eqs. (2.20) and (2.21) define two functions:

$$k^* = M(z^*), \quad z^* = N(k^*), \tag{2.22}$$

which may not be one-to-one or even continuous; that is, $M(\ )$ and/or $N(\ )$ may have several branches and one or more discontinuities. Nonetheless, if we plot these two functions in the k^*–z^* plane, points at which they cross are stationary points. Moreover, the derivatives of these functions may be obtained at any point of continuity along any branch by means of the implicit function theorem. Thus, along a branch

$$\begin{aligned} M' &= \frac{\mathrm{d}k^*}{\mathrm{d}z^*} = \frac{\varphi_z}{1-\varphi_k} = \frac{(1+n^*)g_z + n_z k^*}{1-[(1+n^*)g_k + n_k k^*]}, \\ N' &= \frac{\mathrm{d}z^*}{\mathrm{d}k^*} = \frac{\psi_k}{1-\psi_z} = \frac{(1+n^*)h_k + n_k k^*}{1-[(1+n^*)h_z + n_z k^*]}. \end{aligned} \tag{2.23}$$

Under general circumstances $k^* = 0 = z^*$ is a stationary point. Thus at least one branch of $M(\ )$ and one of $N(\ )$ must begin at the origin.

Our interest is focused on the positive quadrant of the k^*–z^* plane since negative values make no economic sense. In Fig. 2 we have plotted the curves

$$z^* = M^{-1}(k^*), \quad z^* = N(k^*).$$

$M^{-1}(\ )$ is defined for the particular branch of $M(\ )$ starting at the origin; there may be other branches. Both curves start from (0, 0) and are initially increasing (if not increasing there would be no nontrivial stationary point in the positive quadrant for this branch). The curve M_1^{-1} is plotted first increasing, then decreasing. When N_1 has a slope initially less than M_1^{-1} and the latter turns down, we find that a nontrivial stationary point (k_1^*, z_1^*) exists. When N_2 has a slope initially greater than M_1^{-1} and does not decrease, there is no nontrivial stationary point. When M_2^{-1} is not strictly concave, there may be several nontrivial stationary points with different local stability properties. And, of course, if N is not strictly increasing there may be a great many nontrivial equilibria. Furthermore, since several branches of both functions not starting at the

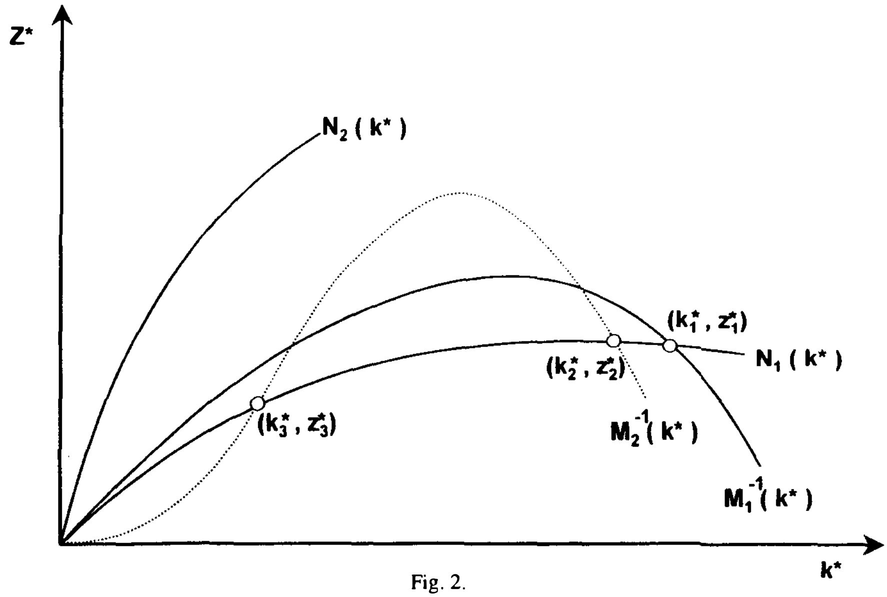

Fig. 2.

origin may exist, these too define stationary points where they intersect. The method discussed there may be used to analyze the dynamic properties of the system. The analysis is, however, complicated and does not, at this level of abstraction, lead to much insight.

The following extended example, however, is helpful in understanding the nature and existence of stationary points: Let the per capita surplus available for investment (and thus, by assumption, per capita investment itself) be

$$s_t = y_t - w_t = k_t f_k(k_t, z_t) + z_t f_z(k_t, z_t), \tag{2.24}$$

and denote functions or values evaluated at a nontrivial stationary point (k^*, z^*) by affixing an asterisk.

Example

Assume a Cobb–Douglas production function

$$y_t = k_t^{\sigma} z_t^{\mu}, \quad 0 < \sigma, \mu; \quad \sigma + \mu < 1, \tag{2.18'}$$

and a linear approximation to $n(w_t)$ which yields

$$1+n(k_t,z_t)=\frac{N_{t+1}}{N_t}=\frac{w_t}{w_m}, \quad w_m>0. \tag{2.19'}$$

Then Eq. (2.20) becomes

$$k_{t+1}=\frac{(1-\delta)k_t+s_t}{w_t/w_m}, \tag{2.20'}$$

where

$$s_t=(\sigma+\mu)y_t$$

and

$$w_t=y_t-s_t=(1-\sigma-\mu)y_t.$$

Hence, the function M^{-1} introduced above is defined explicitly by

$$M^{-1}(k^*)=z^*=\left\{\frac{(1-\delta)(k^*)^{1-\sigma}}{\left[\frac{1-(\sigma+\mu)}{w_m}\right]k^*-(\sigma+\mu)}\right\}^{1/\mu} \tag{2.25}$$

Suppose that $\psi(k_t, z_t)$ is also linear in logs:

$$z_{t+1}=k_t^{\alpha}z_t^{\beta}, \quad 0<\beta<1, \tag{2.21'}$$

but α may be negative. Then

$$N(z^*)=(k^*)^{\alpha/(1-\beta)}. \tag{2.26}$$

If $0<\alpha<(1-\beta)$, that is, if $\alpha+\beta<1$, $N(z^*)$ in Eq. (2.24) is a concave function of z^*, since $0<\beta<1$ is assumed. This assumption might be violated if, for example, Z represented environmental quality, so that per capita environmental quality might be degraded at higher capital–labor ratios.

The shape of $M^{-1}(z^*)$ in Eq. (2.25) depends on the parameters and on their relationship to one another. When, for example, $1/\mu$ is an even number, $M^{-1}(k^*)$ increases from 0 to $+\infty$ as k^* goes from zero to $k_m=w_m(\sigma+\mu)/[1-(\sigma+\mu)]$. For larger values of k^*, $M^{-1}(k^*)$ is a decreasing function of k^*, with

$$\frac{\mathrm{d}z^*}{\mathrm{d}k^*} = \frac{-z^*}{\mu}\frac{\dfrac{\sigma(1-\sigma-\mu)}{w_\mathrm{m}}k^* + (\sigma+\mu)(1-\sigma)}{k^*\left[\dfrac{(1-\sigma-\mu)}{w_\mathrm{m}}k^* - (\sigma+\mu)\right]} < 0 \quad \text{for } k^* > k_\mathrm{m}.$$

Thus, for $k^* > k_\mathrm{m}$, $\alpha, \beta > 0$ and $\alpha + \beta < 1$ and $\sigma, \mu > 0$ and $\sigma + \mu < 1$, there exists a unique nontrivial stationary point. Moreover, this point is a stable equilibrium. It is interesting to note that the location of this equilibrium depends on the value of the minimum wage, w_m. The values of k^* and z^* determine the rate of growth of population from Eq. (2.19′):

$$\frac{N_{t+1}}{N_t} = \frac{[1-(\sigma+\mu)](k^*)^\sigma (z^*)^\mu}{w_\mathrm{m}}$$

$$= \frac{[1-(\sigma+\mu)](k^*)^{\sigma+(\alpha\mu/(1-\beta))}}{w_\mathrm{m}}.$$

If

$$w_\mathrm{m} = [1-(\sigma+\mu)](k^*)^{\sigma+(\alpha\mu/(1-\beta))},$$

population too will be stationary. If w_m exceeds this value, population will be declining (eventually to zero). If w_m is less, population will be increasing. In general, the smaller w_m, the higher the rate of population growth at a stationary point and the lower per capita consumption.

In this example

$$\xi_k = \frac{w_\mathrm{m}}{1-(\sigma+\mu)}\left\{\frac{(1-\delta)(1-\sigma)}{y^*}\right\},$$

$$\xi_z = \frac{w_\mathrm{m}}{1-(\sigma+\mu)}\left\{\frac{-\mu(1-\sigma)}{y^*}\right\},$$

$$\eta_k = \alpha,$$

$$\eta_z = \beta.$$

Thus

$$\operatorname{tr} \boldsymbol{J} = \frac{(w_\mathrm{m}/y^*)(1-\delta)(1-\sigma)}{1-(\sigma+\mu)} - \beta,$$

$$\det \boldsymbol{J} = \frac{(w_\mathrm{m}/y^*)(1-\delta)}{1-(\sigma+\mu)}(\alpha\mu + \beta(1-\sigma)).$$

That is

$$\det J = \left[\frac{\alpha\mu + \beta(1-\sigma)}{1-\sigma}\right] \operatorname{tr} J + \beta\left[\frac{\alpha\mu + \beta(1-\sigma)}{1-\sigma}\right] = A \operatorname{tr} J + B. \tag{2.27}$$

Eq. (2.27) determines a straight line in the tr J–det J plane with slope

$$A = \frac{\alpha\mu}{1-\sigma} + \beta$$

and intercept

$$B = \frac{\alpha\beta\mu}{1-\sigma} + \beta^2.$$

Under plausible assumptions: $0 < \sigma < 1$, $0 < \mu < 1$, $\sigma + \mu < 1$, and $0 < \beta < 1$, but the sign of α is ambiguous.

Consider first the case in which $\alpha > 0$, that is increases in the capital–labor ratio favorably affect the stock of Z. Then A and B are clearly positive. For example, if $\sigma = 0.5$, $\mu = 0.25$, $\alpha = 0.25$, and $\beta = 0.5$, $A = 0.675$ and $B = 0.3125$.

If, on the other hand, $\alpha < 0$, so that an increase in the capital–labor ratio negatively impacts on the stock Z, then the signs of A and B are ambiguous. $A \gtreqless 0$ according as $(\mu/(1-\sigma)) \lesseqgtr (-\beta/\alpha)$. For example, in the previous case suppose $\alpha = -0.25$, then $A = 0.375$ and $B = 0.1875$. But suppose that instead $\beta = 0.25$ and $\alpha = -0.75$, then $A = -0.125$ and $B = -0.03125$.

Plot the line det $J = A$ tr $J + B$ on a background similar to Fig. 1, Nerlove (1993), for positive A and B. The line clearly crosses regions of both stability and instability. Where on this line we are determines whether or not the equilibrium is stable. In the first numerical example

$$\operatorname{tr} J = \frac{(w_m/y^*)(1-\delta)(1-\sigma)}{1-(\sigma+\mu)} - \beta$$

$$= 2\left(\frac{w_m}{y^*}\right)(1-\delta) - \frac{1}{2}.$$

It follows that the issue of stability or instability turns on the magnitude of $(w_m/y^*)(1-\delta)$ and since $(1-\delta)$ is likely to be close to one, on the ratio of the minimum wage to per capita output at the stationary point. Only if this is rather small, will the equilibrium be stable. In particular tr J must be less than $(1-B)/A$, which places a clear constraint on $(w_m/y^*)(1-\delta)$ given the values of α, β, σ and μ.

This extended example shows that the existence and nature of an equilibrium of the capital–labor and resource–labor ratios with endogenous population turn crucially on the minimum wage in relation to output or to the capital–labor ratio. If this is very large, the resulting equilibrium, although likely to exist, will generally be unstable.

2.4. *Lucas and Romer: new directions in growth theory*

Recent work extending Solow's and Swan's earlier contributions focuses on the implications of increasing returns and investment in human capital. Solow (1992) characterizes the neoclassical growth model, i.e. Solow–Swan and its derivatives, as follows: "The main implication of this model is that, no matter where it starts, it tends eventually to a steady state independent of the initial conditions (i.e. the initial stock of capital). In that steady state extensive quantities like aggregate output are growing at a rate equal to the sum of the rates of labor-force growth and labor-augmenting technological progress. Thus per capita output, capital and consumption all grow at the same rate as technology is improving. ...the asymptotic growth rate for the model economy depends only on *the exogenously given rates of technological progress and population growth* [italics supplied]". Modifying the basic Solow–Swan model so as to allow for endogenous population growth and/or endogenous saving does allow for a somewhat richer set of conclusions; in particular the possibility that multiple stationary equilibria exist allows each economy to arrive at a point depending on the initial capital stock and population. It remains true, however, that per capita growth in output, consumption and capital stock can only occur asymptotically at the same rate as technical progress.

Romer (1986, 1990) and Lucas (1988) argue that this conclusion does not accord with even the most basic facts: new technology is widely accessible everywhere with little lag. Thus, even though the actual levels of capital stock, consumption and output per capita may differ depending on initial conditions, *rates of growth* should tend to equality everywhere. If there were only one unique stationary equilibrium, the model predicts that poor countries should grow faster than rich countries but that all should eventually grow at the same rate. Although the facts are in some dispute (see Baily and Schultze, 1990; Mankiw et al., 1992), there is some evidence that it is primarily the poorest countries that continue to fall behind those that entered the modern industrial age by mid-twentieth century, which is clearly at variance with the implication of neoclassical growth theory without endogenous population growth. Rather, however, than seek an explanation in terms of one of several stable equilibria in a model with endogenous fertility and/or mortality, both Lucas and Romer, as well as other contributors who have followed their lead, seek an explanation in terms of endogenous technical change and thus to endogenize the per capita growth rate.

One way to endogenize growth is to augment the neoclassical model so that investment in knowledge, and thus technical change, is economically motivated. Boserup's approach, discussed in the preceding section, represents a variant of this, although without additional assumptions, cannot explain growing per capita consumption or divergencies among countries. The assumption of increasing returns may also provide an avenue, as Young (1928) recognized long ago, although, as Solow (1992) notes increasing returns by itself is not sufficient to achieve endogenous growth. What Lucas does, in addition, is to identify technical progress with the accumulation of "human" capital, but as a stock of knowledge which survives the bearer rather than as a stock of skills which must be embodied in a particular human agent and which disappears when that agent dies. Both Lucas and Romer model production technology so that individual firms see constant returns to scale in the inputs they control but, because of favorable spillovers, there are increasing returns in the aggregate. This is in sharp contrast to the way in which Nerlove (1974) treated human capital in his explanation of the demographic transition; there the central feature of human capital was precisely that it died with the bearer so that an exogenous fall in death rates, particularly infant and child mortality, greatly enhanced the returns to parental investment in the human capital of their children, one form of which were in the form of better health and nutrition, further increasing the probability of survival. Additionally, Lucas assumes that aggregate production technology can be represented as $F(K, HL)$, where K is the stock of physical capital, L is employment (proportional to population), and H is the accumulated stock of "human" capital or of knowledge per capita. That aggregate output depends only on the total stock of "human" capital or knowledge and not on how it is embodied is an extremely strong assumption and one which we would regard as unrealistic: Are ten workmen who can read, for example, always worth one hundred who cannot no matter what the size of the labor force?

Increasing returns are fundamental to the arguments of both Romer and Lucas but in different ways: Romer assumes that the creation of new "knowledge" by one firm has positive external effect on the production possibilities of other firms so that the production of the consumption good as a function of stock of knowledge exhibits increasing returns. Lucas, on the other hand, assumes that individuals acquire productivity enhancing skills by investing time in learning; accumulation of skills by one individual not only enhances his productivity, it also enhances the productivity of all workers through its positive spill-over effect on the average skill level of the whole labor force; "human" capital or the stock of individual skills is produced from accumulated average level of skills and foregone labor time with increasing returns (constant returns to each factor individually) (see Raut and Srinivasan (1993) for an extensive survey of these models).

It should be noted that the spill-over effects of the average stock of human capital per worker in the Lucas model and of knowledge in the Romer model are exter-

> nalities unperceived (and hence not internalized) by individual agents. However, for the economy *as a whole* they generate increasing scale economies even though the perceived production function of each agent exhibits constant returns to scale. Thus by introducing nonconvexities through the device of a Marshallian externality Lucas and Romer are able to work with intertemporal competitive (albeit a socially nonoptimal) equilibrium. Both in effect make assumptions that ensure that the marginal product of physical capital is bounded away from zero, and, as such, it is not surprising that in both models sustained growth in *income* per *worker* is possible. Thus Lucas and Romer avoid facing the problem that research and development (R&D) which leads to technical progress, is naturally associated with imperfectly competitive markets. (Raut and Srinivasan, 1993: p. 8.)

Under either set of assumptions, it is possible to show that steady-state growth rates depend on preferences and on policies affecting private incentives or disincentives to invest in either form of capital. So, while these models are ad hoc, the purpose of explaining why steady-state growth rates do not converge is accomplished. Because of Lucas' treatment of "human" capital as a stock of general knowledge, which lives on after its initial embodiment in an individual human being, and Romer's treatment of a general stock of knowledge, however, their models are of little use in accounting for the demographic transition. What is required for that is a deeper understanding of the incentives of parents to invest in the individual-specific human capital of their children. Growth in general knowledge and endogenous technical change are undoubtedly important to the understanding of the process of economic growth more generally, but families' decisions with respect to the numbers of children they have and those that affect their survival are crucial to understanding how economic growth and demographic change are related.

3. The microeconomics of endogenous population: fertility, mortality and investment in children

In this section, we explore a number of recent models of family decision-making relevant to the relation between demographic change and economic growth. We begin with a discussion of the fundamental trade-off between the quality and the quantity of children. We continue with a survey of models which emphasize parental altruism as the motivation for parental concern with both numbers and quality of children. Next, we turn to nonaltruistic motives for having surviving children such as parental security in old age and discuss models in which parents may invest in their children so as to enhance the chances for their survival or to improve their future productivity. Finally, we return to a discussion of two-sided altruism (parents for children and children for parents) as a basis for parents' transfers to children and children's subsequent transfers to parents.

3.1. Quality versus quantity: the Becker–Lewis model[1]

The trade-offs between quantity and quality of children and between quantity of children and income, explored in this section, were advanced by Becker and Lewis (1973). If parents care about the numbers and welfare of their children and, at least partly, control these variables so as to maximize their own utility, certain nonlinearities and nonconvexities are introduced in the budget constraint which they face, and certain characteristics of the utility function which they maximize are modified in contrast to the constraint and maximand encountered in the traditional theory of consumer choice. The seminal paper of Becker and Lewis (1973) shows how these modifications may affect the relation between income and desired fertility even when children are a normal good.

Consider a pair of parents as an individual decision-maker who consumes units of a single composite consumption good (c). The parents also extract utility from the number of their children (n) and the quality, or well-being (b), of each one of them. This quality is measured by the units of the single composite good spent on these children (e.g., on their education, health, etc.). For the sake of simplicity, we treat n as a continuous variable. We come back to the importance of the discrete variability of n below. In addition, we assume that all children are identical and that the parents treat them symmetrically, so we use the symbol b for the quality of every child.

The parents have a direct utility function,

$$u^*(c, b, n), \tag{3.1}$$

where $u_i^* > 0$, $i = 1,2,3$. This means that the parents extract positive utility from all three variables, c, b, and n. The parents choose both b and n in addition to c. Note that we implicitly assume that parents correctly anticipate that each of their children will be identical and all will be born at the beginning of the decision period. Let the parents' income, in terms of the single composite good, be I. They spend c on themselves and a total of bn on their children. We also allow for a pecuniary benefit from each child, denoted by a and measured in terms of the single composite good. Parents correctly anticipate this benefit, which could be a child allowance paid by the government, a wage earned by the child and contributed to the household income, etc. The benefit could also be negative if there is a tax on children. Thus, the parents' budget constraint is

$$c + bn \leq I + an. \tag{3.2}$$

The term bn makes the budget constraint nonlinear. Furthermore, the budget set $\{(c, b, n) \mid c + bn < I + an\}$, describing the parents' feasible bundles of c, b, and n, is

[1] This section draws on the earlier exposition of Nerlove et al. (1987: Ch. 5).

not convex. Notice, however, that the utility function, Eq. (3.1), can still have all the properties conventionally assumed, such as increasing monotonicity and quasi-concavity.

The analysis and results of traditional theory hold for a linear budget constraint but with some modification can be applied to this case. The basic idea is that, since, as long as the Marshallian and Hicksian demand functions are well defined and differentiable, standard results follow. The analysis is carried out in Nerlove et al. (1987: Ch. 5) by simply redefining the choice variables of the consumer unit so as to obtain a linear budget constraint at the expense of losing the conventional properties of the utility function. Then, the conventional results apply with respect to the newly defined variables.

Specifically, defining by q the total expenditure on children (i.e., $q = bn$) and letting

$$u(c, q, n) = u^*(c, q/n, n), \tag{3.3}$$

the parents' optimization problem is to choose c, q, and n so as to maximize Eq. (3.3) subject to the following linear budget constraint:

$$c + q \leq I + an. \tag{3.4}$$

Observe that while u^* is monotonically increasing in its third argument, n (i.e., $u_3^* > 0$), it follows from Eq. (3.3) that u need not increase in n because of the term q/n in u^*:

$$u_3 = u_3^* - \frac{qu_2^*}{n^2}.$$

Furthermore, the number of children in the budget constraint, Eq. (3.4), appears in the same way as the labor supply appears in a conventional model, i.e., it adds to income rather than to expenditures (assuming $a > 0$). Thus, at the parents' optimum, the marginal utility of n (namely, u_3) must be negative. With this linear budget constraint, all the conventional results of traditional theory hold with respect to the variables c, q, and n.

If the economy, in the context of which the representative household makes its fertility decisions, is growing, the typical family's income will be growing. In the Becker–Lewis model, it is possible that fertility is reduced even though children are a normal consumption good. To see this, consider first the consumer optimization problem in terms of the original utility function, u^*, and the nonlinear budget constraint Eq. (3.2):

$$\max_{c,b,n} u^*(c,b,n), \quad \text{such that } c + bn \leq I. \tag{3.5}$$

(To simplify the analysis and make it comparable to that of Becker and Lewis, we let $a = 0$. $a \neq 0$ will play a further role in multiperiod models in which transfers from children to parents also occur.) One can see that the quality of children (b) is the "price" of the quantity of children (n) and vice versa. Thus, some of the parents' choice variables also act as prices, and the usual conditions on the utility function that guarantee the normality of a certain good do not apply. New conditions must be derived.

The optimal c, b, and n in the problem, Eq. (3.5), all depend on income I. Denote the optimal c, b, and n by $C(I)$, $B(I)$, and $N(I)$, respectively; we are interested here in the sign of the elasticity of N with respect to I. As we noted, this is not the standard question as to whether a certain good is a normal good, and one cannot use the standard conditions for normality. Therefore, we form a hypothetical problem that is a standard consumer optimization problem; we explain below how it is related to our true problem, Eq. (3.5).

Consider the following problem:

$$\max_{c,b,n} u^*(c,b,n), \quad \text{such that } c + p_b b + p_n n \leq I + M, \tag{3.6}$$

where $p_b > 0$, $p_n > 0$, and M are parameters. One can interpret p_b and p_n as the "prices" of quality and quantity of children, respectively; M is interpreted as a lump-sum transfer. Now Eq. (3.6) is a standard consumer optimization problem, and one denotes the optimal bundle of c, b, and n by $\overline{C}(p_b, p_n, I + M)$, $\overline{B}(p_b, p_n, I + M)$, and $\overline{N}(p_b, p_n, I + M)$, respectively. The latter functions are conventional Marshallian demand functions and, in particular, we assume that they exhibit normality:

$$\overline{C}_3, \overline{B}_3, \overline{N}_3 > 0.$$

Comparing Eq. (3.5) with Eq. (3.6), it is straightforward to establish the relationship between (C, B, N) and $(\overline{C}, \overline{B}, \overline{N})$. Evaluated at $p_b = N(I)$, $p_n = B(I)$ and $M = N(I)B(I)$, the bundle $(\overline{C}, \overline{B}, \overline{N})$ is equal to (C, B, N):

$$\begin{aligned} &\overline{C}(N(I), B(I), I + N(I)B(I)) = C(I), \\ &\overline{B}(N(I), B(I), I + N(I)B(I)) = B(I), \\ &\overline{N}(N(I), B(I), I + N(I)B(I)) = N(I). \end{aligned} \tag{3.7}$$

Differentiating totally the last two relationships with respect to I:

$$(\overline{B}_1 + B\overline{B}_3)\frac{\mathrm{d}N}{\mathrm{d}I} + (\overline{B}_2 + N\overline{B}_3 - 1)\frac{\mathrm{d}B}{\mathrm{d}I} = -\overline{B}_3,$$
$$(\overline{N}_1 + B\overline{N}_3 - 1)\frac{\mathrm{d}N}{\mathrm{d}I} + (\overline{N}_2 + N\overline{N}_3)\frac{\mathrm{d}B}{\mathrm{d}I} = -\overline{N}_3. \qquad (3.8)$$

Employing the Hicks–Slutsky equations corresponding to the hypothetical problem, Eq. (3.6), one sees that $\overline{B}_3 + B\overline{B}_3$ is the Hicks–Slutsky substitution effect of the "price" of the quality of children on the quantity of children demanded. Denote this effect by $\overline{S}_{bb}$. Also, $\overline{B}_2 + N\overline{B}_3$ is the Hicks–Slutsky substitution effect of the "price" of the quantity of children on the quality of children demanded: denote it by $\overline{S}_{bn}$. Similarly, $\overline{N}_1 + b\overline{N}_3 = \overline{S}_{nb}$, and $\overline{N}_2 + N\overline{N}_3 = \overline{S}_{nn}$. By the symmetry of the Hicks–Slutsky effects, $\overline{S}_{bn} = \overline{S}_{nb}$. Substituting these relationships into Eq. (3.8) and solving for $\mathrm{d}N/\mathrm{d}I$:

$$\frac{\mathrm{d}N}{\mathrm{d}I} = \frac{\overline{N}_3(1-\overline{S}_{nb}) + \overline{B}_3\overline{S}_{nn}}{(1-\overline{S}_{nb})^2 - \overline{S}_{bb}\overline{S}_{nn}}. \qquad (3.9)$$

In elasticity terms, Eq. (3.9) becomes

$$\eta_{nI} = k\frac{\overline{\eta}_{nI}(1-\overline{\varepsilon}_{nb}) + \overline{\eta}_{bI}\overline{\varepsilon}_{nn}}{(1-\overline{\varepsilon}_{nb})^2 - \overline{\varepsilon}_{bb}\overline{\varepsilon}_{nn}}, \qquad (3.10)$$

similarly,

$$\eta_{bI} = k\frac{\overline{\eta}_{bI}(I-\overline{\varepsilon}_{nb}) + \overline{\eta}_{nI}\overline{\varepsilon}_{bb}}{(1-\overline{\varepsilon}_{nb})^2 - \overline{\varepsilon}_{bb}\overline{\varepsilon}_{nn}}, \qquad (3.11)$$

where

$\eta_{nI} = \dfrac{\mathrm{d}N}{\mathrm{d}I}\dfrac{I}{N}$, income elasticity of $N(I)$,

$\overline{\eta}_{nI} = \overline{N}_3\dfrac{\overline{I} + \overline{NB}}{\overline{N}}$, income elasticity of $\overline{N}(\,)$ (assumed positive),

$\overline{\eta}_{bI} = \overline{B}_3\dfrac{\overline{I} + \overline{NB}}{\overline{B}}$, income elasticity of $\overline{B}(\,)$ (assumed positive),

$k = \dfrac{I}{\overline{I} + \overline{NB}} < 1,$

$$\bar{\varepsilon}_{nn} \equiv \frac{\bar{S}_{nn} p_n}{\bar{N}} = \frac{\bar{S}_{nn}\bar{B}}{\bar{N}},$$ own-substitution elasticity of $\bar{N}(\,)$,

$$\bar{\varepsilon}_{bb} \equiv \frac{\bar{S}_{bb} p_b}{\bar{B}} = \frac{\bar{S}_{bb}\bar{N}}{\bar{B}},$$ own-substitution elasticity of $\bar{B}(\,)$,

$$\bar{\varepsilon}_{nb} \equiv \frac{\bar{S}_{nb} p_b}{\bar{N}} = \bar{S}_{nb},$$ cross-substitution elasticity.

Thus, one can see from Eq. (3.10) that, if there is a unitary substitution elasticity between the quantity and quality of children (i.e., $\bar{\varepsilon}_{nb} = 1$), then $\eta_{nI} = -(k/\bar{\varepsilon}_{bb})\eta_{bI} > 0$, by the negativity of own-substitution elasticity, $\bar{\varepsilon}_{bb}$, and the normality of $b(\bar{\eta}_{bI} > 0)$. In this case an increase in income increases fertility (and, as can be seen from Eq. (3.11), child quality as well).

Now assume that the substitution elasticity between the quantity and quality of children is larger than 1 (i.e., $\bar{\varepsilon}_{nb} > 1$). Also assume that total expenditure on children increases with income (i.e., $N(I)B(I)$ increases in I). This means that at least one of the components of this expenditure, $N(I)$ or $B(I)$, must be increasing in income. Suppose then that $\eta_{bI} > 0$. Since it is assumed that $\bar{\varepsilon}_{nb} > 1$, it follows that the numerator on the right-hand side of Eq. (3.11) is negative. Hence the denominator must also be negative. But it then follows from Eq. (3.10) that η_{nI} is positive. Thus, under the assumption that total expenditure on children increases in income, a high degree of substitutability between child quality and quantity (i.e., $\bar{\varepsilon}_{nb} > 1$) implies that income has a positive effect on both the quantity and the quality of children (i.e., both η_{nI} and η_{bI} are positive).

However, as this quantity–quality problem is not a standard consumer choice problem, one can extract from Eq. (3.10) many cases in which income has a negative effect on fertility (i.e., $\eta_{nI} < 0$). If the substitution elasticity between the quantity and quality is smaller than one ($\bar{\varepsilon}_{nb} < 1$), there are two possibilities.

One possibility is that the denominator of Eq. (3.10) or Eq. (3.11) is positive. This occurs when the own-substitution elasticities ($\bar{\varepsilon}_{bb}$ and $\bar{\varepsilon}_{nm}$) are relatively low. In this case one can see from Eq. (3.10) that if the income elasticity of quality in the hypothetical problem, Eq. (3.6) (namely, η_{bI}) is substantially higher than the income elasticity of quantity in the same problem (namely, η_{nI}), then child quantity falls with income ($\eta_{nI} < 0$) while child quality rises ($\eta_{bI} > 0$).

The other possibility is that the denominator of Eq. (3.10) or Eq. (3.11) is negative. This occurs when the own-substitution elasticities ($\bar{\varepsilon}_{bb}$ and $\bar{\varepsilon}_{nm}$) are relatively high. In this case, if $\bar{\eta}_{bI}$ is substantially lower than $\bar{\eta}_{nI}$, then, again, $\eta_{nI} < 0$ and $\eta_{nI} > 0$.

Introducing the quantity and "quality" of children in parents' utility function introduces a nonlinearity and nonconvexity in the budget constraint. However, a reformulation in which the budget constraint is linear but the utility function is no longer monotonically increasing and strictly quasi-concave permits us to apply the conventional theory of consumer choice to derive the result that, even if the income elasticities of demand for both quantity and quality of children are positive, the observed (uncompensated) elasticity of fertility (numbers of children) with respect to income may be negative. Whether or not this occurs depends in part on the elasticity of substitution between quantity and quality of children in parents' utility function.

3.2. Parental altruism and investment in human capital

In a series of papers, Becker (1988), Becker and Barro (1988), Barro and Becker (1989), and Becker et al. (1990, extended in Tamura, 1992), seek to establish a connection among fertility, bequests which may be in the form of human capital formation, parental altruism and economic growth. In their basic model, parents choose both the number of children and the capital, both human and physical, bequeathed to each child. One possible interpretation of the "quality" variable in the Becker–Lewis formulation, presented in Section 3.1, is as the investment or bequest in the form of human capital to the child. Parents' choices are driven by the trade-off between the altruism which they feel towards their children and the satisfactions which they derive from their own consumption and from having children. More formally, parents choose the optimal values of their own consumption, the number of children and the capital transferred to each child, taking into account the costs of rearing children and the dependence of their own utility on the utility of their children. This analysis thus represents an extension of the Becker–Lewis model of the trade-off between quality and quantity of children. The difference between the model discussed here and the earlier model is that the "quality" of children is now given an explicit interpretation in terms of bequests to children in the form of human and/or physical capital, and an explicit rationale for the inclusion of such a "quality" measure in parental utility functions is given in terms of the "love" which parents feel towards their children, i.e., the degree to which children's utilities enter parents' utilities and trade-off with parents' own consumption and desires for children.

In this formulation there is no explicit recognition of the reasons why even basically altruistic parents might want surviving children. Although many would not regard such an omission as a serious shortcoming (need we offer any reason for the enjoyment of a symphony?), even in economically developed societies parents derive obvious benefits from surviving children in old age, and concern with old age security is an even more important consideration in poor countries with limited means for parents to transfer consumption from productive and healthy years to years of decline. In the next section of this part, we consider a model in which parents have children and invest in them both because they "love" them and because they expect benefits from their children in old age. This formulation raises an extremely difficult issue as to why parents might expect their children to contribute to parents' welfare at the expense of their own consumption and contribution to their own children's welfare. The answer is presumably "reverse altruism", that concern of children for their parents' welfare. But, as we shall see, certain asymmetries, across generations backwards versus forwards, complicate the analysis, as well as divergencies among individual endowments which may result when survival is stochastic.

The aim of the Barro and Becker (1989), analysis is to describe the way in which the economy and population evolve through time in consequence of the endogeneity of fertility *and* capital formation. Becker (1988), for example, is quite explicit about this aim, in describing research designed to model economies which have both low-welfare equilibria with high fertility or population and low levels of capital and those with low fertility and high levels of capital and/or economic growth. Extension of microeconomic models of fertility and human and physical capital formation in this direction requires, however, certain general equilibrium considerations.

Suppose that parents care about the number of their children and their children's welfare or utility. Employing the notation of the model presented above but adding the generational subscript t, we now have first-generation utility

$$u_t = u(c_t, n_t, u_{t+1}). \tag{3.12}$$

If parents and children do not differ each from the other and from one another,

$$u_{t+1} = u(c_{t+1}, n_{t+1}, u_{t+2})$$

and so on. Moreover, each child receives the same bequest from her parent. (Suppose one-parent families.) Imagine this bequest, b_t, to be in the form of physical capital and to act simply as an addition to the endowment of each child, i.e., each individual in the next generation. Thus the problem of each parent is to maximize u_t in Eq. (3.1) subject to the budget constraint

$$c_t + (b_t + a)n_t \leq I_t + b_{t-1}, \tag{3.13}$$

where a denotes additional exogenous costs (or benefits) of rearing a child. This is an extremely difficult problem and not one which, as far as we know, is capable of any general solution. However, Becker and Barro (1988) (see also Nerlove et al., 1987: p. 78; Barro, 1974; Razin and Ben Zion, 1975) assume a particular form of additively separable utility:

$$u_t = v(c_t, n_t) + \beta(n_t)n_t\hat{u}_{t+1}. \tag{3.14}$$

$\beta(n_t)$ measures the degree of altruism per child which Becker and Barro (B-B) assume to be decreasing with n, $\beta' \leq 0$. We have put a hat over the utility of the next generation to indicate that it is the parent's "estimate" of each child's utility. B-B assume perfect foresight and replace $\hat{u}_{t+1}$ with u_{t+1}, but this is quite a leap, although, of course, if one does take perfect foresight seriously, it is a natural starting point. Somewhat different and not uninteresting conclusions emerge if $\hat{u}_{t+1}$ is some function of the *parent's* choices of b_t and n_t. In this case, $\hat{u}_{t+1}$ is the maximum taken over c_{t+1}, b_{t+1} and n_{t+1}, given I_t, b_t and the expectation (or estimate) of per child endowment, $\hat{I}_{t+1}$, but it is not necessarily the same as u_{t+1}.

3.2.1. A nonrecursive formulation with altruism and bequests

Let us define $f_t(b_t, \hat{I}_{t+1}) = \hat{u}_{t+1}$ to be the parent's expectations of her child's future welfare. Then the parent's problem is

$$\max_{c,n,b}\{v_t(c_t, n_t) + \beta(n_t)n_t f_t(b_t, \hat{I}_{t+1})\},$$

$$\text{such that } c_t + (b_t + a)n_t \leq I_t + b_{t-1}, \tag{3.15}$$

where I_t, $\hat{I}_{t+1}$ and b_{t-1} are given. Neglecting the integer restriction on n_t and assuming an interior solution, the first-order conditions are

$$\begin{aligned} v_1 &= \lambda, \\ v_2 + \beta(n_t)[1 - \varepsilon_\beta]f(b_t, \hat{I}_{t+1}) &= \lambda[b_t + a], \\ \beta(n_t)n_t f'(b_t, \hat{I}_{t+1}) &= \lambda n_t, \end{aligned} \tag{3.16}$$

where $\varepsilon_\beta = -n\beta'/\beta$ the elasticity of the degree of altruism with respect to the number of children. These conditions have the following interpretation: λ is the marginal utility of the parent's consumption. The elasticity ε_β is defined as positive since $\beta' \leq 0$ is assumed. If $\beta' = 0$, $\varepsilon_\beta = 0$ and β is a constant, equivalent in the B-B formulation to the usual form in which the future is discounted. If $\varepsilon_\beta < 1$, there is a positive benefit from having an additional child in addition to the direct marginal utility v_2 which results from the altruistic impact of the child's welfare. These benefits, in turn, are set equal to the marginal costs of the child valued in terms of the parent's own consumption, $\lambda[b_t + a]$. These costs are not parameters of the problem because they depend on the parent's choice of bequest, through λ, and on the choice of her own consumption level, as well. Finally, cancelling n_t from both sides of the last equation, the marginal benefit of increasing the bequest, which is solely altruistic, is equated to the marginal utility of consumption. If $u_t = u[c_t, n_t, b_t] = v_t(c_t, n_t) + \beta n_t f(b_t, \hat{I}_{t+1})$ is assumed to be a concave function of the three arguments, the conditions Eq. (16) are necessary and sufficient for an interior maximum. The problem is identical to the one which we analyzed above with $z_t = b_t + a$ and I replaced by $I_t + b_{t-1}$.

While, for the reasons adduced above, increases in exogenous endowment or previous parent's bequest need not increase fertility n_t, other effects may be less ambiguous: an increase in the exogenous cost of a child unambiguously increases the cost of having an additional child and therefore results in substitution of bequests and own consumption for child numbers, i.e., a reduction in fertility. What about the effect of an exogenous increase in the degree of altruism? Let β be constant. An increase in β, holding bequests, child numbers and the parent's expectations constant, unambiguously increases the parent's utility. While such an increase in altruism may increase both child numbers or bequests at the expense of the parent's own consumption, fertility may not increase for the same reason that an increase in the exogenous endowment of the parent need not increase fertility even if both child numbers and their welfare are normal goods.

Note that parental bequests act only through parent's altruism and their effects on parent's expectations of the welfare of her offspring. Eliminate altruism, and bequests are eliminated. Assuming minimal effects on parental expectations of her ability to influence the welfare of her children, standard theory applies with respect to the effects of changes in child-rearing costs and in exogenous parental endowment. But, even in this case, it is interesting to note that parents will unambiguously have more children than if there were no altruism (because of the additional utility generated by the term $\beta n f(\hat{I}_{t+1})$ in the utility function) despite the fact that they leave no bequests.

3.2.2. *A recursive formulation with altruism and bequests*

The formulation of the preceding subsection does not lend itself to the analysis of growth paths of population and bequests because parents' behavior depends on their expectations of the future welfare of their children, which only partly depend on the bequest left to each. Change parental expectations and you change everything. In a deterministic world, rational expectations and perfect foresight coincide. If in addition, children have the same utility functions, then under the separability assumption of the previous section, the recursive utility function (3.12) yields the following *dynastic utility function* of the parent in period 0:

$$\sum_{t=0}^{\infty}\left[\prod_{\tau=0}^{t}\gamma(n_{\tau-1})\right]v(c_t,n_t) \quad \text{where } \gamma(n_t)=\beta(n_t)n_t \text{ and } \gamma(n_{-1})\equiv 1, \tag{3.17}$$

where the constraint set involves equality. In order for the infinite sum in Eq. (3.17) to converge, it suffices that $v(.)$ is bounded and

$$0<\beta(n_t)n_t<1 \tag{3.18}$$

for the sequence in question. Clearly this will not be the case for all sequences. If we further assume that the children have the same exogenous endowments (independent of parental bequests) as their parents do, the counterpart of the maximization problem of the previous section becomes

$$\max_{\{c_t, n_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty}\left[\prod_{\tau=0}^{t}\gamma(n_t)\right]v(c_t, n_t),$$

$$\text{such that } c_t + (b_{t+1} + a)n_t = I + b_t, \quad t \geq 0. \tag{3.19}$$

We have replaced the inequality in the constraint of problem (3.15) by equality, since it will never be optimal to leave anything over if children and their welfare and consumption are all desirable.

Under certain conditions, the problem in Eq. (3.19) can be solved using dynamic programming techniques. For all $t \geq 0$, let $u^*(b_t)$ be the value function of generation t for given level of bequest, b_t, that a parent of generation t received from his/her parents. Then the Bellman equation or functional equation of the problem in Eq. (3.19) is

$$u^*(b_t) = \max_{c,n,b_{t+1}}\{v(c_t, n_t) + \beta(n_t)n_t u^*(b_{t+1})\},$$

$$\text{such that } c_t + (b_{t+1} + a)n_t = I + b_t. \tag{3.20}$$

The problem is then to solve for a differentiable concave value function $u^*(.)$ that satisfies Eq. (3.20). Given this value function, from the first-order conditions of problem (3.20) one obtains the optimal solution for n_t and b_{t+1} (and hence c_t) as a function of b_t.:

$$b_{t+1} = g(b_t), \quad n_t = h(b_t). \tag{3.21}$$

Usually, one starts with the formulation in Eq. (3.17) and derives Eq. (3.20), and characterizes the solutions (see Stokey et al. (1989: pp. 66–102) for an exposition of this technique). A serious problem in our context of endogenous fertility is that even when we assume that the utility function $v(c_t, n_t)$ and the degree of altruism $\beta(n_t)n_t$ are all concave, the value function defined in Eq. (3.20) is not concave. Assuming Eq. (3.22) below, Benhabib and Nishimura (1989) within the above framework, and Nishimura and Raut (1993) in a somewhat more general framework, characterize the local dynamics of the optimal solution of Eq. (3.20) around a convex neighborhood of a steady state in which the optimal value function u^* is differentiable and concave.

B-B (1988, 1989), Becker et al. (1990), and Tamura (1992), for example, all restrict that part of the additively separable utility function of the parent referring to the parent's *own* utility from consumption and children, to depend on the parent's consumption alone, that is

$$v_n = 0. \tag{3.22}$$

Even then the analysis is too difficult to be carried out explicitly.

Two further restrictions introduced in B-B (1988) are:

(a) Constant elasticity altruism:

$$\beta(n_t) = \beta_0 n_t^{-\beta_1}.$$

(b) Constant elasticity utility of own consumption:

$$v(c_t) = c_t^{\sigma}, \quad 0 < \sigma < 1.$$

Two extensions to the formulation above designed to make the micro model adaptable to an equilibrium growth framework are:

(c) Replace I by w_t, a variable adult wage rate.

(d) Replace b_t by $(1 + r_t)k_{t-1}$, where k_t is the per capita stock of physical capital bequeathed to each child and r_t is the rate of return to physical capital.

The parent's budget constraint in Eq. (3.19) then becomes

$$c_t + (k_t + a)n_t = w_t + (1 + r_t)k_{t-1} \tag{3.23}$$

and his/her problem to maximize the so-called *dynastic utility* function:

$$u_0(c_0, n_0, c_1, n_1, \ldots) = \sum_{t=0}^{\infty}\left[\prod_{\tau=0}^{t}\gamma(n_{\tau-1})\right]c_t^{\sigma}, \tag{3.24}$$

subject to constraint (3.23). The variables controlled by the parent are his/her own consumption, c_t, the number of children he/she has, n_t, and his/her bequest per child of physical capital, k_t. Variables exogenous to the parent's decision are the adult wage, w_t, his/her parent's bequest, k_{t-1}, and the rate of return on physical capital, r_t. The cost of rearing a child, a, and the elasticities $-\beta_1$ and σ, and the coefficient β_0 are also parameters of the parent's problem.

One of the underlying assumptions in these models is that the parent perfectly foresees the paths of future w_t and r_t (and, of course, his/her children as parents do so too). We leave it to the reader to judge how realistic such a perfect foresight assumption is, but it is essential to the development which follows.

For the existence of a solution with a positive number of children, the condition

$$\sigma < 1 - \beta_1 \tag{3.25}$$

is necessary, which can be seen as follows. Let

$$N_i = \prod_{t=0}^{i-1} n_t$$

be the number of descendants in the ith generation of an individual alive now. $C_i = N_i c_i$ = total consumption in the ith generation. Then, since

$$\begin{aligned} u_0 &= c_0^\sigma + \beta_0 n_0 n_0^{-\beta_1} c_1^\sigma + \beta_0^2 (n_0 n_1)(n_0 n_1)^{-\beta_1} c_2^\sigma + \cdots \\ &= c_0^\sigma + \beta_0 N_1^{1-\beta} c_1^\sigma + \beta_0^2 N_2^{1-\beta} c_2^\sigma + \cdots \\ &= c_0^\sigma + \beta_0 N_1^{1-\beta_1-\sigma} (N_1 c_1)^\sigma + \cdots, \end{aligned}$$

it follows that

$$\frac{\partial u_0}{\partial N_i} = (1 - \beta_1 - \sigma) \frac{\beta_0^i N_i^{1-\beta_1-\sigma} (N_i c_i)^\sigma}{N_i}.$$

Hence, Eq. (3.25) is a condition that $\partial u_0 / \partial N_i > 0$, a condition which must hold near the maximizing values of n_1 and k_1 if parents are to produce children at all.

Neglecting integer restrictions, as we have throughout this section, B-B obtain the first-order conditions by setting the derivatives of the Lagrangian

$$u_0^* \equiv \sum_{i=0}^{\infty} \{ c_i^\sigma \beta_0^i N_i^{1-\beta_1} - \lambda_i [c_i + (k_i + a) n_i - w_i - (1 + r_i) k_{i-1}] \} \tag{3.26}$$

equal to zero:

$$\begin{aligned} \frac{\partial u_0^*}{\partial c_j} &= \frac{\sigma V_j}{c_j} - \lambda_j = 0, \\ \frac{\partial u_0^*}{\partial n_j} &= \sum_{i=j+1}^{\infty} \frac{(1-\beta_1) V_i}{N_i} \frac{\partial N_i}{\partial n_j} - \lambda_j (k_j + a), \\ &= \frac{1}{n_j} \sum_{i-j+1}^{\infty} (1-\beta_1) V_i - \lambda_j (k_j + a) = 0, \\ \frac{\partial u_0^*}{\partial k_j} &= -\lambda_j n_j + \lambda_{j+1} (1 + r_{j+1}) = 0, \quad j = 1, 2, \ldots, \end{aligned} \tag{3.27}$$

where $V_j = \beta_0^j N_j^{1-\beta_1} c_j^{\sigma}$. There is also a "dynastic" budget constraint based on the fact that the individual budget constraints must hold each period:

$$k_0 + \sum_{i=0}^{\infty} \frac{N_i w_i}{\prod_{j=0}^{i}(1+r_j)} = \sum_{i=0}^{\infty} \frac{[N_i c_i + N_{i+1} a]}{\prod_{j=0}^{i}(1+r_j)}. \tag{3.28}$$

Taking ratios of successive equations in Eq. (3.27) yields three intertemporal "arbitrage" conditions:

$$\begin{aligned} \frac{\lambda_{j+1}}{\lambda_j} &= \frac{n_j}{1+r_{j+1}} \\ &= \frac{V_{j+1}}{V_j}\frac{c_j}{c_{j+1}} \\ &= \frac{n_j(k_j+a)\sum_{i=j+2}^{\infty} V_i}{n_{j+1}(k_{j+1}+a)\sum_{i=j+1}^{\infty} V_i}. \end{aligned} \tag{3.29}$$

Since

$$\frac{V_{j+1}}{V_j} = \beta_0 n_j^{1-\beta_1}\left(\frac{c_{j+1}}{c_j}\right)^{\sigma},$$

these equations can be rewritten in terms of the "discounted" value of the ratios of consumption in successive periods and the ratio of total costs (bequests plus rearing cost) of children:

$$\frac{\lambda_{j+1}}{\lambda_j} = \frac{n_j}{1+r_{j+1}} = \beta_0 n_j^{1-\beta_1}\left[\frac{c_{j+1}}{c_j}\right]^{\sigma-1}. \tag{3.30}$$

The ratio of the birthrate to returns to physical capital $(1 + r_{j+1})$ provides the key intertemporal link between the trade-off between the total costs of having children in different generations and per capita consumption of parents in each generation. Given an initial bequest endowment in the first generation, Eqs. (3.29) or (3.30) determine the law of motion of birthrates and bequests, and, since the constraint (3.20) holds in each period, of consumption per capita, as functions of rearing costs, wage rates and returns to capital. Coupled with an economy-wide production function which would determine wage rates and returns to capital as functions of total population and total capital stock, the B-B model provides a complete endogenous explanation with only initial wealth, k_0, and rearing costs, a, as exogenous. Indeed, such an explanation is B-B's aim in B-B (1989). We note here some of the implications of changes in wages, rates of return to capital, rearing costs and altruism on the behavior of fertility, bequests and consumption. Some of these implications are sensitive to the functional form of the utility function and the way in which altruism enters.

1. Consumption per capita, c_j, rises across generations only if the rearing cost a rises, and does not depend on β_0, the degree of altruism (time preferences), on fertility, nor on the return to capital, r_j (interest rates).
2. Changes in the returns to capital affect mainly fertility, n_j. This variable increases with the interest rate and with the degree of pure altruism.
3. Changes in initial wealth, k_0, do not affect future consumption per person if child rearing costs do not change. Greater wealth affects initial consumption of the "dynasty" head but also results in increased fertility which offsets this effect for future generations. Nor are future bequests per child affected by changes in initial wealth.
4. A tax on children in the jth generation, compensated by an increase in initial wealth, increases consumption and reduces fertility only in the jth generation (if the return to capital is unaffected). Even a permanent compensated tax on children reduces fertility only in the generation enacted, but, of course, total population in successive generations is lower as a result even though its rate of growth is unchanged.
5. Constancy of w, r, and a over time and the particular form of the altruism function and utility function ensure that a unique steady state of fertility, consumption and bequests, *across* generations, exists and is globally stable. Such a state is reached in one generation starting from any initial position. Note, however, that such a steady state is not a steady state of the economy in general equilibrium.

The model in B-B (1988) is the basis for the discussion in B-B (1989), but in Becker et al. (1990) it is modified so that the stock of a parent's human capital affects the time costs of rearing children and market wages. In turn, stocks of human capital affect the relative desirability of investing in the human capital of one's children and bequests in the form of physical capital. The 1990 model is primarily directed to an economy-wide explanation of fertility and per capita consumption and relies heavily on the assumption that increases in the total stock of human capital in the economy lead to increased rather than decreased returns.

3.3. Survival probability, fertility and investment in health care

In the Becker–Lewis model, parents value the number of their offspring and their "quality" as measured by the bequests parents leave their children, but there is no hint in the formulation about why parents might be concerned with the number of children they have. Willis (1980) suggests that a major motive for having children in less developed economies may be to provide for old-age security (see also Nerlove et al., 1987: Ch. 9). In this subsection we explore the model of Sah (1991) and its extension by Dalko (1992), in which fertility choices result from a dependence of parents' utility on the number of *surviving children*, because, for example, the consumption of parents when they are old depends on the number of their offspring who are around to support them. Why this should be so, and explicitly what the trade-off between present consumption and future consumption (number of surviving children) is, is not considered until the following subsection. The focus of Sah's model is on the discrete nature of both births and surviving children and on the stochastic nature of the latter. Dalko focuses on the way in which parents can influence survival probabilities by investments in their children.

3.3.1. The basic model

Let n be the number of children born in an individual family (a choice variable) and N be the number who survive to adulthood. N is a random variable which is assumed to follow a binomial distribution for given n and survival probability s, which we assume to be known to the family and exogenous to its choice. The ex ante costs of a birth, $C(n)$, are assumed to be nondecreasing and a concave function of n. Ex post costs and benefits are summarized in the parents' utility function, $u(N)$, which is a concave function of N, first increasing and then decreasing, and assumed to account for the costs of raising surviving children and the effect of increased numbers on parents' current consumption. Parents, who are not altruistic in the sense of caring about their children's future welfare, are assumed to maximize the expected utility of births, given the survival probability s:

$$\max_{n} U(n,s) = \sum_{N=0}^{n} b(N,n,s)u(N), \tag{3.31}$$

where

$$b(N,n,s) = \binom{n}{N} s^{N}(1-s)^{n-N}$$

is the probability that exactly N children survive from n births.

$U(n, s)$ is a discrete function of the discrete variable n and a continuous function of the continuous variable s. The binomial density is "bell-shaped" although discrete, and its values are approximated by the ordinates of a normal density with the same mean,

ns, and variance, $ns(1-s)$. The expectation of $u(N)$ is obtained from the binomial distribution conditional on n, the number of births, so it has both a larger mean and a larger variance for a higher n, given the survival probability, s. Since $u(N)$ is "parabolic" in shape, the effect of convoluting it with $b(N, n, s)$ is to "flatten" it out and, as long as $u(N)$ remains positive, to move more mass into the upper tail: thus, if $u(N)$ is rising, $U(n, s)$ will also be rising but at a lower rate. For given n, the mean of the binomial increases but the variance rises or falls according as $s \leq 1/2$ or $s \geq 1/2$; thus, for $s \geq 1/2$ more weight is given to values of $u(N)$ above and near to the previous mean, $U(n, s)$.

One further rather drastic simplification of the Sah model is that the number of births is determined once and for all at the beginning of the decision period and not sequentially. Sequential determination makes quite a difference, as Wolpin (1984) shows.

While it is natural to focus on infant and child mortality as Sah's model does, in the real world maternal mortality, particularly in child birth (or in abortion-averted births), is extremely important. To the extent that $C(n)$ reflects these factors, ex ante costs are a much more integral part of the analysis than may appear.

If $U(n, s)$ and $C(n)$ were continuous functions of n, the optimum number of births would be obtained by equating the marginal utility of an additional birth, U_n, to its marginal cost, C_n. Conditions for the existence of a positive optimum number of births are apparent in this case: U_n is falling, eventually becoming negative after $U(n, s)$ reaches its maximum, so as long as C_n is nondecreasing and does not exceed U_n at $n = 0$, a positive optimum exists. If we had been dealing with continuous functions of a continuous variable, we would simply differentiate this optimum with respect to s in order to discover the effects of increasing survival probability on the number of births. Unfortunately nothing so simple is possible in this case.

Provided there is a maximum number of children attainable, it is possible to formulate a discrete analog to the usual first-order conditions which determine the optimal number of births, $\hat{n}$, as a *discontinuous* function of the survival probability s. Sah (1991) proves the following propositions about this function:

Proposition 1: *$\hat{n} = n(s)$ is either unique or there are at most two neighboring values for the same value of s.*

Proposition 2: $\partial n(s)/\partial s \leq 0$.

He proves additionally that parents' utility is nondecreasing in s, that is parents are no worse off and possibly better off if infant and child mortality declines:

Proposition 3: $\partial U(\hat{n}, s)/\partial s \geq 0$.

While Proposition 3 is not astonishing, its implication for Proposition 2 is a little surprising. Although a fall in infant and child mortality makes parents better off and, therefore, if children are a normal good, might be expected to increase the demand for surviving children and thus to offset partially the negative effects on births of greater survival probabilities, it does not do so in this case as long as the number of births is near the optimum simply on account of the way in which s enters the binomial coefficients in Eq. (3.1). The result has nothing to do with the shape of the ex post utility function. Consequently, if the independence of surviving births were not assumed or if ex ante costs were not assumed to be separable, this rather strong conclusion would not necessarily be obtained.

3.3.2. Implications for population growth

Notwithstanding the qualifications which must be attached to Sah's Proposition 2 that increasing survival probabilities result in a fall in the optimal number of births, there is considerable empirical evidence to support such a relationship (see Freedman, 1975; Preston, 1978; Schultz, 1981). But to say that *fertility* falls with increasing survival probability is not the same as the proposition that the *rate of growth* of population declines with falling infant and child mortality. Indeed, in general, it does not do so at all levels of survival probability, as shown in Nerlove (1991).

In one of the models developed there, it is assumed that survival probability is the only exogenous factor influencing parental decisions with respect to fertility, and it is also assumed that, together with its effects on mortality, it is the only exogenous factor influencing the rate of growth of population. Of course, survival probability may be influenced in part by family decisions or by macroeconomic conditions influencing the availability of food or other environmental factors. Abstracting from such complications, let births per family be a decreasing function of s, the survival probability. If all families were identical one-parent households with exactly the same perceptions of survival probability, then aggregate behavior would also be discrete. On the other hand, if family preferences differ and/or perceptions of survival probabilities, then under appropriate assumptions we can treat the aggregate function as continuous and differentiable. Let the total number of parents in the population in period t be N_t. If births per parent in the tth generation are n_t, then

$$\frac{N_{t+1}}{N_t} = sn_t \tag{3.32}$$

in the aggregate, provided N_t is large. Suppressing the subscript t, let

$$\rho(s) = sn(s) \tag{3.33}$$

be the rate of growth of the aggregate population. Then

$$\rho' = n\left(1 + s\frac{n'}{n}\right). \tag{3.34}$$

Although $n' < 0$, $0 \leq \rho' \leq 0$ according as the elasticity of births with respect to survival probability is less than, or greater than one in absolute value. In terms of the preceding analysis, whether this is the case depends on factors affecting the shape of the ex post utility function beyond concavity or lack of it with respect to the number of surviving children. It is argued in Nerlove (1991), however, that even if the elasticity of births is greater than one in absolute value in relatively favorable regimes of infant and child mortality, it must nonetheless approach zero as survival probabilities decline to very low levels simply because of the biological maximum to the number of children a women can have. We might also suppose that ex ante costs of child bearing would also increase in regimes of high mortality. Thus, at very low levels of survival probability, the rate of growth of population will generally *increase* as a result of falling infant and child mortality even though the number of births per parent declines. When survival probabilities become high, the fall in the optimal number of births as infant and child mortality declines further will more than offset the larger numbers of such births which survive.

3.3.3. Investment in health care and survival probability

Without altruism, the only incentive parents have to invest in their children is to increase their own consumption in old age. More surviving children will augment parents' consumption but so will an increase in the endowment each surviving child is able to attain. The preceding analysis can be extended simply by dividing $c_{0,t}$ into two parts:

$$c_{0,t} = h_t + k_t, \tag{3.35}$$

where h = health care investment and k = investment in other forms of children's human capital. Ex ante costs of births and survival probability become functions of h and e_{t+1} becomes a function of k. Because

$$c_2 = \mu N e_{t+1}(k),$$

we see there are two ways to influence expected old-age utility, through increasing the expected number of survivors and by increasing the endowment of each. The division of c_0 into two parts thus requires obvious modifications in the first-order conditions, breaking them into two and replacing $c_{0,t}$ by $h_t + k_t$ in the constraint. In general, after allowing for the marginal effect, e_{t+1}, on a surviving child's endowment, the conditions require simultaneous equality *at the margin* of the expected old-age utility of an increase per birth in health care, affecting survival, and the other investment, affecting a surviving child's endowment, with the marginal disutility of the necessary decrease in parents' consumption as young adults. These marginal utility rates of return to the two forms of investment depend on the responsiveness of the survival probability, on the one hand, and the child's endowment, on the other, to the two different forms of investment. Both affect young–adult consumption, but health care expenditures may have a partially offsetting effect by reducing ex ante birth costs.

In other respects the analysis closely parallels that given in the preceding subsection. The possibility of influencing child survival and the earnings capacity of one's offspring means that increases in parents' endowments or those of their children, which make parents better off, may lead to decreases in fertility because such improvements in parents' welfare lead them to spend more on their children, which, in turn, increases the probability that their children will survive. Similarly, a small exogenous upward shift in the survival function may lead to either higher or lower rates of population growth.

3.4. *Transfers from children to parents and fertility: old-age security motive*

So far we have considered models in which motivation for children, and bequest in the form of health, nutrition and education and physical capital are assumed to be either parental concern for their children's welfare or for the number of surviving children. We have examined the effects of survival probabilities, child rearing cost and other factors on the growth rates of population and income. Some of the studies along these lines also examine the interaction between household decisions and the aggregate economy. In this section, we survey a similar literature that developed almost independently of the above. In this literature, the decisions regarding children, and investment in their skill, health and nutrition are motivated by the amount of transfers that parents can obtain from such investments in their old-age. It is apparent that such decisions will depend on the mechanism by which intergenerational transfers are made and that these decisions are affected by the existence of publicly provided transfer mechanisms such as pay-as-you-go social security program, or subsidies to children's education, health and nutrition. We begin with a summary of a few models of endogenous fertility and growth in which the transfer mechanism from children to parents is assumed to be determined by social norms; we point out what bearing these optimizing models have on the nature of dynamics of the models we discussed in Section 1; we then discuss how these models could be more appropriately integrated with the previous models of this section by introducing two-sided altruism and discuss the associated technical difficulties.

3.4.1. The old-age pension motive for children

When the capital markets are missing or imperfect, parents may treat their children analogously to capital goods, i.e., as a vehicle for transferring consumption from present to future. Although many authors have pointed out this possibility, and Leibenstein (1957, 1974) attempted partial microeconomic analysis of fertility demand, Neher (1971) was the first to model formally the old-age security hypothesis and study its consequences on aggregate population and income growth. We consider a simplified version of Neher's model.

3.4.1.1. Ethics of equal sharing within household

Neher (1971) considers an agrarian economy of overlapping generations, all living in an extended family and having a certain plot of land. Let L_t denote the number of adults in the family in period t. Adult members of the family work on the family land and produce food C_t that is shared equally among all members. Assume that food cannot be stored. The food production in period t ($t \geq 0$) is given by

$$C_t = F(L_t).$$

$F(.)$ is assumed to exhibit first increasing and then decreasing marginal product of labor. Let n_t be the number of children that an adult of period t decides to have. All adults are assumed identical. Those living in period t are denoted by the superscript t. With this convention, we denote the parent's consumption in youth in period $t-1$ by c^t_{t-1}, the parent's consumption in adulthood in period t by c^t_t and the parent's old-age consumption in period $t+1$ by c^t_{t+1} The equal sharing rule means that for given L_{t-1}, L_t and n_{t+1}, the constraints, Eq. (3.36) and Eq. (3.37) below, hold. Thus the problem of the adult head of family of period t is to

$$\max U(c^t_t, c^t_{t+1}) = \alpha u(c^t_t) + \beta u(c^t_{t+1}), \tag{3.36}$$

subject to

$$\begin{aligned} c^t_t &= \frac{F(L_t)}{L_t(1+n_t) + qL_{t-1}}, \\ c^t_{t+1} &= \frac{F(L_t n_t p)}{L_t n_t p(1+n_{t+1}) + qL_t}, \end{aligned} \tag{3.37}$$

where p is the survival probability of children to adulthood, and q is the survival probability from adulthood to old-age. In the above utility function, β may be regarded as discount factor adjusted for the probability of old-age survival. Assuming that $\lim_{c \to 0} u'(c) = \infty$, then clearly $n_t > 0$ for all $t \geq 0$. Thus the first-order condition for the above problem after simplification becomes

$$\frac{\alpha u'(c_t^t)c_t^t L_t}{L_t(1+n_t)+qL_{t-1}} = \frac{\beta u'(c_{t+1}^t)[F'(L_t n_t p)]pL_t}{pL_t n_t(1+n_{t+1})+qL_t}. \tag{3.38}$$

Notice that it is not possible to derive the dynamics of this economy from Eq. (3.38), since for all $t \geq 0$, the equation can determine only n_t for given n_{t+1}, and there is no other equation that can determine n_{t+1}. Neher restricts his analysis to steady state. A *steady state* of an economy is a situation when all per capita variables are constant over time. Let N_t be the total population in period t. It is apparent that in a steady state

$$L_t = L^* \text{ and } c_t^t = c_{t+1}^t = c^* \text{ and } \frac{L_t}{N_t} = \frac{p}{1+p+pq}$$

for all $t \geq 0$. The steady-state consumption is thus given by

$$c^* = \frac{F(L^*)}{N^*} = \frac{F(L^*)L^*}{L^*\ N^*} = \frac{F(L^*)}{L^*}\frac{p}{1+p+pq}. \tag{3.39}$$

Let us denote a steady-state competitive equilibrium solution with a tilde. It can be shown that the steady-state equilibrium is given by

$$F'(\tilde{L}^*) = \frac{(\alpha+\beta p)\tilde{c}^*}{\beta p}. \tag{3.40}$$

The size of the adult population $\hat{L}^*$ that yields the maximum possible consumption in the steady state is said to be *golden rule of fertility*. From Eq. (3.39) it follows that $\hat{c}^*$ is maximized when average product is maximized. Let us denote the golden rule consumption as $\hat{c}^*$. Given the nature of production function, average product is maximized when the average product is equal to the marginal product of labor. Thus golden rule $\hat{L}^*$ satisfies

$$F'(L^*) = \frac{F(L^*)}{L^*} = \frac{1+p+pq}{p}. \tag{3.41}$$

It is easy to note that the golden rule of fertility and the competitive equilibrium level of fertility in the steady state are both given by $\hat{n}^* = \tilde{n}^* = 1/p$. Thus higher infant mortality leads to higher fertility in the steady-state in this model.

Let us adopt the golden rule as our optimality criterion as did Neher. Comparing Eqs. (3.40) and (3.41), it is obvious that the steady-state competitive equilibrium results in:

$$\begin{bmatrix} \text{over} \\ \text{optimal} \\ \text{under} \end{bmatrix} \text{population according as } \frac{\alpha+\beta p}{\beta p} \begin{bmatrix} < \\ = \\ > \end{bmatrix} \frac{1+p+pq}{p}. \tag{3.42}$$

Since the agents cannot fully internalize the consequences of their decisions, their decisions are not socially optimal. Let us take an extreme case by assuming that the agents do not discount their future utilities, i.e., $\alpha = \beta = 1$; then from Eq. (3.41) it is clear that competitive equilibrium always results in overpopulation. The same result is true when agents discount future utilities very little, i.e., for β close to one and $\alpha = 1$. In fact, given mortality rates, there exists $\beta_* < 1$ such that the competitive equilibrium will result in overpopulation for all $\beta_* < \beta \leq 1$.

Equal-sharing ethics implicitly assumes that all household members live in a commune and do not try to break out of the sharing principle. If we further assume that an agent can inherit a share of the family land only if he or she stays in the household, and that there are no other assets that can transfer consumption from working age to old-age, it is reasonable to assume the equal sharing principle. However, if there are other assets such as physical capital, gold or paper money, then some agents may be better off by breaking out of the joint family transfer arrangements and instead of depending on their children for an old-age pension, they might prefer to accumulate other assets for old-age support.

We do observe that household members leave their families and move to cities for better opportunities and yet send remittances to their old parents. Even if they stay in the rural areas, we observe that joint family structure often breaks down and new atomistic household units are formed, and yet children continue to transfer income to their old parents. Some of these limitations, which Neher (1971) also pointed out, have been rectified in recent growth models based on old-age security hypothesis. For instance, Willis (1981) replaces the equal sharing assumption in Neher's framework with the assumption that adult children transfer a fixed amount of income to their old parents. This is more realistic and does not presume that parents live with their children to get this old-age support. However, the determination of the amount of transfer remains unspecified. Willis does not have capital accumulation in his model. Raut (1985, 1991, 1992a) and Bental (1989) allow accumulation of physical assets. While Bental assumes perfect capital markets, Raut assumes imperfection in the capital markets and studies the effect of making capital markets perfect and also examines the long-run effects on population growth, capital accumulation and income distribution of various policies (Raut 1991, 1992a). In the next subsection, we review a framework with imperfect capital markets and study the dynamic consequences of old-age security motives for children and then study the effects of various macro-economic policies.

3.4.1.2. Imperfect capital markets and old-age security hypothesis

Consider a model of overlapping generations in which a member of each generation lives for three periods, the first of which is spent as a child in the parents' household. The second period is spent as a young person working, having and raising children, as well as accumulating capital. The third and last period of life is spent as an old person in retirement living off support received from each of one's offspring and from the sale of accumulated capital. All members of each generation are identical in their preferences defined over their consumption in their working and retired periods. Thus, in this model the only reason that an individual would want to have a child is the old-age support that the child will provide during the parent's retired years.

Assume that technology is characterized by a constant-returns-to-scale production function $Y_t = Z_t F(K_t, L_t)$ which uses capital K_t and labor L_t to produce output Y_t in period t, $t \geq 0$, Z_t is the level of Hicks-neutral total factor productivity in period t. For the moment we assume that $Z_t = 1$ for all $t \geq 0$. We adopt the convention that the producer borrows from the households K_t amount of the $(t-1)$th period aggregate good and promises to pay $(1+r_t)K_t$ amount of the tth period aggregate good. Each adult of period t supplies one unit of labor. We also assume for simplicity that capital depreciates in one generation. Denote the average product of labor by $f(k) \equiv F(k, 1)$. Assume perfect competition in all markets. Profit maximization by producers yields

$$w_{t+1} = f(k_{t+1}) - k_{t+1} f'(k_{t+1}), \tag{3.43}$$

$$1 + r_{t+1} = f'(k_{t+1}). \tag{3.44}$$

Formally, a typical individual of the generation which is young in period t has n_t children, consumes c_t^t and c_{t+1}^t in periods t and $t+1$, and saves s_t in period t. The parent supplies one unit of labor for wage employment. The individual income from wage labor while young in period t is w_t and that is the only income in that period. A proportion a_t of this wage income is given to parents as old-age support. We assume for now that a_t is exogenously given. Later we consider mechanisms determining a_t. While old in period $t+1$, the parent sells accumulated saving to firms and receives from each of the parent's offspring the proportion a_{t+1} of his/her wage income. The parent enjoys a utility $U(c_t^t, c_{t+1}^t)$ from consumption. Thus the parent's choice problem can be stated as

$$\max_{s_t, n_t > 0} U(c_t^t, c_{t+1}^t) \tag{3.45}$$

subject to

$$\begin{aligned} c_t^t + \theta_t n_t + s_t &= (1 - a_t) w_t, \\ c_{t+1}^t &= (1 + r_{t+1}) s_t + a_{t+1} w_{t+1} n_t, \end{aligned} \tag{3.46}$$

where θ_t is the output cost of rearing a child while young, $1 + r_{t+1}$ is the rate of interest between period t and $t + 1$ and w_t is the competitive wage rate in period t.

Note that, in equilibrium, the private rates of returns from investing in children and physical capital must be equal in order to rule out arbitrage, which implies that

$$\frac{a_{t+1} w_{t+1}}{\theta_t} = 1 + r_{t+1}. \tag{3.47}$$

Substituting Eqs. (3.43) and (3.44) in Eq. (3.47), we obtain an implicit relation among k_{t+1}, θ_t and a_{t+1}. It can be shown that under standard neoclassical assumptions on the production function, we can solve this implicit function uniquely to obtain $k_{t+1} = \Psi(\theta_t / a_{t+1})$. Since it is assumed that capital depreciates fully in one generation, $k_{t+1} = s_t/n_t$ and the budget constraints Eq. (3.43) and Eq. (3.44) become respectively

$$c_t^t = (1 - a_t) w_t - S_t \tag{3.43'}$$

and

$$c_{t+1}^t = (1 + r_{t+1}) S_t, \tag{3.44'}$$

where $S_t = [\theta_t + \Psi(\theta_t / a_t)] n_t$. S_t may be thought of as total savings. Denote the solution of this utility maximization problem by $S_t = H(w_t, 1 + r_{t+1})$. We can now express the solutions for n_t and s_t as

$$n_t = \frac{H(w_t, 1 + r_{t+1})}{\theta_t + \Psi(\theta_t / a_t)} \quad \text{and} \quad s_t = \frac{H(w_t, 1 + r_{t+1})}{\Psi(\theta_t / a_t)\, [\theta_t + \Psi(\theta_t / a_t)]}. \tag{3.48}$$

Child rearing involves parents' time which we specify by assuming that $\theta_t = \theta + \eta w_t$ where $\eta > 0$ is the fraction of parents' time spent in rearing each child. We simplify by assuming that $a_t = a$ for all generations. From Eq. (3.47), we have

$$\frac{w(k_{t+1})}{f'(k_{t+1})} = \frac{\theta + \eta w(k_t)}{a}, \tag{3.49}$$

where $w(k) \equiv f(k) - kf'(k)$. Under the assumption that $f(k)$ is strictly concave and satisfies the Inada condition, it can be shown that the left-hand side of Eq. (3.49) is a strictly increasing function of k_{t+1} which goes to zero when k_{t+1} tends to zero and goes to infinity as k_{t+1} tends to infinity. Hence for given k_t there exists a unique k_{t+1}, which leads to a first-order nonlinear difference equation

$$k_{t+1} = \Phi(k_t). \tag{3.50}$$

Eq. (3.50) determines the dynamics of the economy, for once we know the series k_t, we know the series r_t and w_t from Eqs. (3.43) and (3.44) and the series n_t and s_t from Eq. (3.48). Since this is a first-order difference equation, using well-known techniques (Devaney, 1989), the global dynamics of this system may be analyzed. Applying the implicit function theorem to Eq. (3.49), it can be easily shown that

$$\frac{\mathrm{d}k_{t+1}}{\mathrm{d}k_t} = \frac{\eta k_t f''(k_t)[f'(k_{t+1})]^2}{af(k_{t+1})f''(k_{t+1})} > 0.$$

Thus a positive shock in the capital–labor ratio and hence in the per capita income in period t will have positive effects on the capital–labor ratios and per capita incomes in all future periods. Furthermore, a higher child rearing cost, θ_t, in period t or a lower transfer from children, a_t, results in a higher capital–labor ratio and thus per capita income in the next period.

In general we do not know how k_t, s_t and n_t behave over time. It is, however, interesting to note that if $\eta = 0$ we can see immediately from the above that k_t will jump to steady state in period $t = 1$. The dynamic properties of these variables, assuming Cobb–Douglas utility and production function, are considered in the following example.

Example

Assume that utility and production functions are of Cobb–Douglas form:

$$U(c_t^t, c_{t+1}^t) = \alpha \log c_t^t + (1-\alpha)\log c_{t+1}^t, \quad 0 < \alpha < 1,$$

$$f(k) = k^\sigma, \quad 0 < \sigma < 1.$$

Eq. (3.49) simplifies to

$$k_{t+1} = \frac{\sigma\theta}{a(1-\sigma)} + \frac{\sigma\eta}{a} k_t^\sigma. \tag{3.51}$$

From the above it easily follows that $\Phi(k_t)$ is an increasing concave function and for large k_t, $\Phi(k_t) < k_t$. Thus the capital–labor ratio will behave exactly as in the Solow–Swan growth model: there exists a unique globally stable steady-state capital–labor ratio, $k^* > 0$. Moreover, it can be easily shown that

$$\frac{\partial k^*}{\partial \theta} > 0, \quad \frac{\partial k^*}{\partial a} > 0,$$

and that

$$n_t = \left[\frac{(1-a)(1-\alpha)(1-\sigma)a}{a(1-\sigma)+\sigma}\right]\left[\frac{w(k_t)}{\theta + \eta w(k_t)}\right]. \tag{3.52}$$

Since $s_t = \Phi(k_t)n_t$ note that once the dynamic path for k_t is known, we can determine the dynamic paths of n_t and s_t. Moreover, it can be easily shown that $dn_t/dk_t > 0$. It follows that if $k_0 < k^*$, then both k_t, n_t, and s_t will be growing over time and in the long-run they converge to their respective steady-state values.

In Eq. (2.7) of the Solow–Swan model and Eq. (2.16′) of Niehans' model the specifications of the $n(k)$ function were arbitrary and did not help us very much in simplifying the dynamics of the underlying economies. The form in Eq. (3.52) is, however, the result of optimizing individual behavior in the aggregate economy and this form of $n(k)$ leads to simpler dynamics of the underlying economy.

Raut (1985, 1991) uses a more general formulation of the above basic framework in which parents simultaneously choose savings in physical capital, number of children, and investment in their skills. In this model, investment in human capital of children is motivated by old-age transfers as contrasted with the altruistic motives for such investments which we considered earlier. Under certain assumptions it is shown that in general equilibrium the low-skill parents tend to have larger number of low-skill children and no savings in physical capital. Lower-skilled workers earn lower wages in general equilibrium. At the aggregate level this provides different explanation for the commonly observed negative relationship between quality and number of children and that between income and number of children of households.

One of the implications of the old-age security hypothesis is that when a publicly funded pay-as-you-go social security program is introduced in an economy with imperfect capital markets, fertility will decline. Nerlove et al. (1987: Ch. 9), on the other hand, show in a two-period model of old-age security without a capital market that, if parents care about the welfare of their children, then introduction of capital markets may increase the general equilibrium fertility rate due to an income effect strong enough to outweigh the substitution effect.

There have been a number of empirical attempts to examine the effect of introducing publicly provided social security on fertility levels. Although many of these studies suffer from lack of appropriate data to test such an hypothesis, general consensus is there are negative effects on fertility (for a summary, see Nugent (1985), Raut (1991)). The literature on the US and other developed countries focuses mainly on the effect of social security on savings and ambiguous effects have been found. Very little empirical evidence is available on the joint effect of social security on fertility and savings, and on the welfare of different generations. The above framework and its extension have been used to study the long-run effects on income distribution, population and income growth rates of various income redistribution policies such as lump-sum tax transfers, subsidies to human capital of children of unskilled parents, introduction of a pay-as-you-go social security program, and making the capital markets more perfect; see Raut (1991).

The main policy conclusions which emerge from the aforementioned studies are that the dynamic effects of introducing a pay-as-you-go social security program, within the framework described, are that both fertility and saving will decline in the short run and long run, and furthermore, if the percentage of voluntary old-age transfers, a, is smaller than a threshold level, then introduction of such a social security program is welfare-enhancing for all agents in the present and all future generations. Similar conclusions are reached by Bental (1989) and, in a framework with parental altruism, by Nishimura and Raut (1992). Furthermore, income transfers to reduce intra-generational income inequality cause higher income gaps for the children in subsequent generations; if such a redistributive scheme persists over time, then the economy will end up with higher population growth, and a lower rate of capital accumulation in the long run. On the other hand, subsidies to the unskilled parents for the purpose of investing in their children's skills, or introduction of a social security program will lead to slower population growth, lesser income inequality and a higher rate of capital accumulation and intergenerational social mobility.

In the Malthus–Boserup model of Section 2, we introduced a third factor of production, technological knowledge, together with labor and capital. Because our specifications were at the aggregate level, we could not reduce below two the dimension of the underlying dynamic system of the economy. Optimizing models can sometimes simplify the study of dynamics by reducing the model to a lower dimension. Raut and Srinivasan (1994) use the basic framework of this section and assume that as a result of conglomeration and congestion effects of population density on productivity level, population size affects Z_t; however, this effect is treated as Marshallian externality by the individual optimizing agents. They find that the cost of child rearing, θ_t, and the nature of dependence of Z_t on L_t, determine the dynamics of the competitive equilibrium path. For instance, when the child rearing cost is constant, i.e., $\eta = 0$, or when cost involves only the time cost, i.e., $\theta = 0$, the dynamics of the economy reduce to one dimension. The nonlinear dynamics of the model nonetheless generate a plethora of outcomes (depending on the functional forms, parameters and initial conditions); these include not only the neoclassical steady state with exponential growth of population with constant per capita income and consumption, but also growth paths which do not converge to a steady state and are even chaotic. Exponential, and even super-exponential growth of per capita output are possible in some cases.

3.5. *Two-sided altruism and transfers from children to parents*

So far we have assumed that the inter vivos transfers from children to parents are exogenously given or that children simply "tithe" for reasons of custom to support their aged parents. Recently attempts have been made to motivate transfers from children to parents as utility maximizing behavior and how it interacts with fertility and savings decisions (see Srinivasan, 1988; Cigno, 1991; Nishimura and Zhang, 1992; Raut, 1992b). In the basic overlapping-generations framework of this section, Nishimura and Zhang and Srinivasan assume that agents care not only for their own life-cycle consumption but also for their parents' old-age consumption. It is argued in Raut

(1992b) that while this framework provides a motivation for children to transfer part of their incomes to their parents as old-age support, the theory of fertility choice based on such utility functions is incomplete since parents will have no motive to have children, if for example there is a social security program which transfers the amount that the children were voluntarily transferring to their parents.

In contrast to Becker and Barro and Becker et al., discussed above, assume that all transfers from parents to children are inter vivos transfers in the first period of life designed to augment the child's consumption in youth. We begin with a simpler model where all transfers from adults to their aged parents are also inter vivos, solely for the purpose of augmenting the old people's consumption. We allow savings for retirement during productive years. All children born are assumed to survive. We also assume, more restrictively, that the young adult applies the same utility function to the consumption of his/her offspring, his/her aged parent and himself/herself. That is, we assume that there is no difference in the utility of consumption when young or old. Then, if $u(c)$, $u' > 0$, $u'' < 0$, is the utility attached to consumption at level c by anyone in any period of life, the aggregate utility of an individual of generation t is

$$V_t = \delta(n_{t-1})u(c_t^{t-1}) + \alpha u(c_t^t) + \beta u(c_{t+1}^t) + \gamma(n_t)u(c_{t+1}^{t+1}). \tag{3.53}$$

An adult of period t earns wage income w_t in the labor market and *expects* to receive a bequest $b_t \geq 0$ from his/her parents. These two sources of income constitute his/her budget during adulthood. Rearing cost per child in period t is $\theta_t > 0$ units of the period t good. Given his/her adulthood budget, he/she decides the amount of savings $s_t \geq 0$, the number of children $n_t \geq 0$, the fraction of income to be transferred to his/her old parents $a_t \geq 0$; in the next period, he/she retires and expects to receive $a_{t+1}n_t w_{t+1}$ amount of gifts from his/her children, earns $(1 + r_{t+1})s_t$ as return from his/her physical assets, and decides the amount of bequest $b_{t+1} \geq 0$ to leave for each of his/her children. Moreover, agent t's tth period decisions, (a_t, n_t, s_t), overlap with his/her parent's bequest decision, b_t; similarly, his/her bequest decision, b_{t+1}, overlaps with the children's gift decisions, a_{t+1}. The effects of agent t's action, $\alpha^t = (a_t, n_t, s_t, b_{t+1})$ on the levels of his/her own life-cycle consumption and the levels of consumption of his/her parents and children in the periods that overlap with his/her life cycle, depend on his/her parent's action, α^{t-1}, and his/her children's action, α^{t+1}, as follows:

$$c_t^t + s_t + \theta_t n_t = (1 - a_t)w_t + b_t, \tag{3.54}$$

$$c_{t+1}^t + n_t b_{t+1} = (1 + r_{t+1})s_t + a_{t+1}w_{t+1}n_t, \tag{3.55}$$

$$c_t^{t-1} = (1 + r_t)s_{t-1} - n_{t-1}b_t + a_t w_t n_{t-1}, \tag{3.56}$$

$$c_{t+1}^{t+1} = (1 - a_{t+1})w_{t+1} + b_{t+1} - s_{t+1} - \theta_{t+1}n_{t+1} \quad \text{and} \quad c_t^t, c_{t+1}^t > 0. \tag{3.57}$$

Most authors use open-loop Nash equilibrium to characterize equilibrium choices in the above framework (see Fudenberg and Tirole (1991) for all the game theoretic concepts used in this section). Note that there may exist various types of Nash equilibria. In one type, intergenerational transfers may be from children to parents in all periods; refer to such an equilibrium as "gift" equilibrium. In another type, the transfers may be from parents to children in all periods; refer to such an equilibrium as a "bequest" equilibrium. There may be other types of equilibria in which transfers are from children to parents in one period and from parents to children in other periods. As has been pointed out in Raut (1992b), the set of Nash equilibria is in general indeterminate for a given economy; however, steady-state equilibria are always determinate, although they may be multiple.

There are examples of economies in which there exist only two equilibria, one with zero savings and the other with positive savings. At both equilibria, the transfers are from children to parents. Furthermore, the equilibrium with positive savings is characterized by lower levels of fertility, transfers from children and welfare levels than the equilibrium with a zero savings rate.

Since there are multiple Nash equilibria, agents have no clue a priori which of the two equilibria will materialize; this brings a difficult problem of coordinating agents' expectations and thus renders a serious weakness of rational expectations to explain observed behavior.

Open-loop Nash equilibria have serious deficiencies in characterizing adequately the incentives of, and describing the behavior of, economic agents. More specifically, an open-loop Nash equilibrium assumes that each agent takes the actions of other agents as given. At such an equilibrium there may be scope for agents to manipulate their children's behavior to extract more transfers from them. For instance, since parents make their consumption and fertility decisions prior to their children's, parents may find it strategically advantageous to consume more when adult and save little so that they can extract maximum transfers from their children. In addition, the Nash equilibrium concept does not deal with agents' behavior out of equilibrium. Raut (1992b) argues that a sequential game framework and the use of subgame perfection are most appropriate in this context because subgame-perfect equilibrium takes into account behavior or reactions of agents out of equilibrium. Azariadis and Drazen (1993) suggest an alternative nonsequential bargaining framework.

The framework presented here is useful in explaining why transfers from children to parents are observed in many economies, and why the amount of transfers declines with the introduction of public-transfer policies; why a pay-as-you-go social security program exists, and whether it is possible for the current living generations to legislate a pay-as-you-go social security benefit scheme for the current and all future generations such that the policy is time-consistent, i.e., the future generations will have no incentives to amend it; and whether such programs lead to Pareto optimality. This framework is also useful for investigation of the strategic aspects of bequests in the form of health, nutrition and education as well as physical capital; if followed through, the resulting theory could be integrated into a general equilibrium framework to examine what effects there may be on the pattern of the demographic transition, and economic growth.

4. Concluding remarks

Our aim in developing growth models with endogenous population is principally to explain why, as income grows, both birth and death rates fall, and stocks of both human and physical capital per capita increase over time. If multiple stationary paths exist, under what conditions does the economy approach one with a low rate of population growth, and high levels of capital per capita and well-being, and under what the opposite? At present, the state of our knowledge is far from complete.

Review of existing neoclassical growth models with endogenous, but unspecified, population growth, suggests a wide variety of possibilities: multiple equilibria, some of which may be stable and others unstable, characterized by large populations with low per capita incomes or by small populations with high per capita income. Without knowing how population change is related to the stocks of capital, the level of population and other state variables of the economy, however, there is little to distinguish which path may be followed. In a market economy, parents choose freely how many children to have, how much to invest in them in the form of health, nutrition and education, and how much to bequeath to them in the form of claims to physical capital. Rates of return to different forms of investment, taxes and subsidies, and incomes all constrain parents' choices which are made in order to maximize their utilities which depend on their preferences for children and consumption in various periods of life and on their concern for the future well-being of their offspring. The problem is thus to relate the relevant rates of return to family decisions, on the one hand, and to the stocks and other state variables of the economy, on the other. Thus, the interaction of household and economy, acting through rates of return and constrained by production technology and by taxes and subsidies, determines which path the economic-demographic system will follow.

Our survey of recent developments in the literature of the "new home economics" reveals various bits and pieces of the complete model we seek. Becker and Lewis have shown us how the quality and quantity of children interact to allow the possibility of a negative income elasticity for child numbers even though children, in a more general sense, are a normal good. Such an outcome, however, depends on the elasticity of substitution between numbers and quality in parents' preferences being sufficiently high. The plausibility or implausibility of this being the case depends on how quality is interpreted and on a more precise specification of why children are valued by parents.

In attempting to account for a large and increasing proportion of bequests in the form of human capital as well as declining fertility over time, Becker and Barro and co-workers focus on parental altruism. Their analysis falls short, however, in failing to account for how parental expectations about, and perceptions of, their children's future welfare are determined. Their assumption of perfect foresight is implausible for an economy out of equilibrium. And, in order to resolve the question of what path the economy will follow and to which of several possible equilibria from any given initial conditions, it is necessary to characterize out-of-equilibrium behavior.

Next, we turn to recent work of Neher, Willis, and Sah, extended by Dalko, which deals with parents' motivations for having surviving children. The principal conclusion of this analysis is to demonstrate under what conditions improvements in survival probability will result in lower fertility and how parents might trade off investments to enhance survival probabilities of children for numbers. The possibility that a small exogenous change in child and infant mortality might set off a cumulative process of declining mortality and fertility is revealed.

Finally, recent work of Raut and others dealing with the old-age security motive for having children is discussed. The focus in this work is to examine what effects the public policies such as introduction of social security, improvement of capital markets, subsidization of poor's education and lump-sum tax transfers from rich to the poor have on population growth, income inequality and income growth both in the short run and long run. While endogenous determination of transfers from children to parents and from parents to children by introducing two-sided altruism within a sequential framework with subgame perfection as an equilibrium concept is useful for analyzing the incentives that agents face while deciding the number of children and the amount of transfers to children and parents, for analyzing the way in which capital markets and the social provision of old-age security interact with fertility decisions, and also for analyzing out-of-equilibrium behavior, the exercise is often technically insurmountable. Further research along these lines is needed.

In general, accumulating stocks of human and physical capital and of population might be expected ceteris paribus to reduce the respective rates of return, as well as to equalize them, and thus to reduce incentives to invest in these forms of capital, to have children, and to slow the rates of growth of per capita incomes. We do not, at this stage, know whether initial conditions matter, to what extent different societies would be driven to a common or to different equilibria, or whether stable equilibria are likely to be characterized by high levels of well-being and low rates of population growth or the opposite. Answers to these questions remain to be discovered in future research by us or by our readers.

References

Azariadis, C. and A. Drazen (1993), "Endogenous fertility in models of growth", Revista de Análisis Económico 8: 131–144.

Baily, M. and C. Schultze (1990), "The productivity of capital in a period of slower growth", Brookings Papers on Economic Activity (Microeconomics), pp. 369–406.

Bairoch, P. (1976), "Agriculture and the industrial revolution, 1700–1914", in: C.M. Cipolla, ed., The Fontana economic history of Europe, Vol. III (The Harvester Press, London) pp. 452–506.

Barro, R.J. (1974), "Are government bonds net wealth?", Journal of Political Economy 82: 1095–1117.

Barro, R.J. and G.S. Becker (1989), "Fertility choice in a model of economic growth", Econometrica 57: 481–501.

Beaver, S.E. (1975), Demographic transition theory reinterpreted: an application to recent natality trends in Latin America (Lexington Books, Lexington, MA).

Becker, G.S. (1988), "Family economics and macro behavior", American Economic Review 78: 1–13.

Becker, G.S. and R.J. Barro (1988), "A reformulation of the economic theory of fertility", Quarterly Journal of Economics 103: 1–25.

Becker, G.S. and H.G. Lewis (1973), "On the interaction between the quantity and quality of children", Journal of Political Economy 81: 279–288.

Becker, G.S., K.M. Murphy and R. Tamura (1990), "Human capital, fertility and economic growth", Journal of Political Economy 98: S12–S37.

Benhabib, J. and K. Nishimura (1989), "Endogenous fluctuations in the Barro–Becker theory of fertility", in: A. Wenig and K.F. Zimmermann, eds., Demographic change and economic development (Springer-Verlag, Berlin).

Bental, B. (1989), "The old age security hypothesis and optimal population growth", Journal of Population Economics 1: 285–301.

Caldwell, J.C. (1982), Theory of fertility decline (Academic Press, New York).

Cigno, A. (1991), Economics of the family (Clarendon Press, Oxford).

Coale, A.J. (1974), "The history of the human population", Scientific American, Reprinted in: The human population, a Scientific American Book (W.H. Freeman, San Francisco, CA) pp. 12–25.

Dálkó, V. (1992), "Endogenous fertility and human capital investment", Unpublished Ph.D. dissertation (Department of Economics, University of Pennsylvania, Philadelphia, PA).

Devaney, R.L. (1989), An introduction to chaotic dynamical systems, 2nd edn. (Addison-Wesley, Reading, MA).

Freedman, R. (1975) The sociology of human fertility (Irvington Publishers, New York).

Fudenberg, D. and J. Tirole (1991), Game theory (The MIT Press, Cambridge, MA).

Kreps, D.M. (1990), Game theory and economic modelling (Clarendon Press, Oxford).

Lee, R.D. (1986), "Malthus and Boserup: a dynamic synthesis", in: D. Coleman and R. Schofield, eds., The state of population theory: forward from Malthus (Basil Blackwell, Oxford) pp. 96–130.

Leibenstein, H. (1957) Economic backwardness and economic growth (Wiley, New York).

Leibenstein, H. (1974), "An interpretation of the economic theory of fertility: promising path or blind alley?", Journal of Economic Literature 12: 457–479.

Lucas, R.E., Jr. (1988), "On the mechanics of economic development", Journal of Monetary Economics 22: 3–42.

Malthus, T.R. (1798), An essay on the principle of population (J. Johnson, London).

Malthus, T.R. (1830), A summary view of the principle of population (John Murray, London).

Mankiw, N.G., D. Romer and D.N. Weil (1992), "A contribution to the empirics of economic growth", Quarterly Journal of Economics 107: 407–437.

Neher, P. (1971), "Peasants, procreation and pensions", American Economic Review 61: 380–389.

Nerlove, M. (1974), "Household and economy: toward a new theory of population and economic growth", Journal of Political Economy 82: S200–S218.

Nerlove, M. (1991), "Population and the environment: a parable of firewood and other tales", American Journal of Agricultural Economics 73: 1334–1357.

Nerlove, M. (1993), "Procreation, fishing and hunting: problems in the economics of renewable resources and dynamic planar systems", American Journal of Agricultural Economics 75: 59–71.

Nerlove, M., A. Razin and E. Sadka (1987), Household and economy: welfare economics of endogenous fertility (Academic Press, New York).

Niehans, J. (1963), "Economic growth with two endogenous factors", Quarterly Journal of Economics 77: 349–371.

Nishimura K. and L.K. Raut (1992), "Family expansion and capital accumulation of a dynasty", Discussion paper no. 92-40 (University of California-San Diego, San Diego, CA), October.

Nishimura K. and J. Zhang (1992), "Pay-as-you-go public pensions with endogenous fertility", Journal of Public Economics 48: 239–258.

Nugent, J.B. (1985), "The old-age security motive for fertility", Population and Development Review 11: 75–97.

Preston, S.H. (1978), The effects of infant and child mortality on fertility (Academic Press, New York).

Raut, L.K. (1985), "Three essays on intertemporal economic development: essay II", Unpublished Ph.D. dissertation (Yale University, New Haven, CT).

Raut, L.K. (1991), "Capital accumulation, income distribution and endogenous fertility in an overlapping generations general equilibrium model", Journal of Development Economics 34: 123–150.

Raut, L.K. (1992a), "Effects of social security on fertility and savings: an overlapping generations model", Indian Economic Review 27: 25–43.

Raut, L.K. (1992b), "Subgame perfect manipulation of children by overlapping generations of agents with two-sided altruism and endogenous fertility", Discussion paper no. 92-54 (University of California-San Diego, San Diego, CA), December.

Raut, L.K. and T.N. Srinivasan (1993), "Theories of long-run growth: old and new", in: K. Basu, M. Majumdar and T. Mitra, eds., Capital, investment and development (Basil Blackwell, New York).

Raut, L.K. and T.N. Srinivasan (1994), "Dynamics of endogenous growth", Economic Theory 4: 777–790.

Razin, A. and U. Ben Zion (1975), "An intergenerational model of population growth", American Economic Review 69: 923–933.

Romer, P. (1986), "Increasing returns and long run growth", Journal of Political Economy 94: 1002–1037.

Romer, P. (1990), "Endogenous technical change", Journal of Political Economy 98: 71–102.

Sah, R.K. (1991), "The effects of child mortality changes on fertility choice and parental welfare", Journal of Political Economy 99: 582–606.

Schultz, T.W. (1961), "Education and economic growth", in: N.B. Henry, ed., Social forces influencing American education (University of Chicago Press, Chicago, IL).

Schultz, T.W. (1971), Investment in human capital (Free Press, New York).

Schultz, T.W. (1973), "Explanation and interpretations of the increasing value of human time", Woody Thompson Lecture (Midwest Economics Association, Chicago, Chicago, IL).

Schultz, T.P. (1981), Economics of population (Addison-Wesley, Reading, MA).

Solow, R.M. (1956), "A contribution to the theory of economic growth", Quarterly Journal of Economics 70: 65–94.

Solow, R.M. (1992), "New directions in growth theory", Unpublished manuscript.

Srinivasan, T.N. (1988), "Fertility and old-age security in an overlapping generations model", Journal of Quantitative Economics 4: 11–17.

Stokey, N.L., R.E. Lucas, Jr. and E.C. Prescott (1989), Recursive methods in economic dynamics (Harvard University Press, Cambridge, MA).

Swan, T.W. (1956), "Economic growth and capital accumulation", Economic Record 32: 324–361.

Tamura, R. (1992), "Fertility, human capital and the wealth of families", Unpublished working paper (Department of Economics, University of Iowa, Iowa City, IA).

Willis, R.J. (1980), "The old-age security hypothesis and population growth", in: T. Burch, ed., Demographic behavior: interdisciplinary perspectives on decision making (Westview Press, Boulder, CO).

Wolpin, K.I. (1984), "An estimable dynamic stochastic model of fertility and child mortality", Journal of Political Economy 92: 852–874.

World Bank (1992), Development and the environment, World development report 1992 (Oxford University Press, New York).

Young, A.A. (1928), "Increasing returns and economic progress", Economic Journal 38: 527–542.