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ENDOGENOUS FERTILITY, TECHNICAL CHANGE AND GROWTH IN A MODEL OF OVERLAPPING GENERATIONS

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Abstract

The consequences of private reproduction and capital (physical and human) accumulation decisions to long-run economic development have been the focus of recent research. The earlier literature on the rate of growth of population, labour force and human capital were assumed to be exogenous. The recent literature, in contrast, explicitly recognizes their endogeneity. In addition, greater emphasis is placed on human as contrasted with physical capital in the growth process.

Another strand of recent literature, labeled as "new" growth theory, is based on a misleading characterization of traditional neoclassical growth theory, namely, that it shows the steady state growth rate of income to be exogenous, and will equal the rate of growth of the labour force in the absence of exogenous technical change. Thus in the steady state per worker output and consumption are constant. A goal of 'new' theory is essentially to endogenize growth and to obtain sustained growth in per worker output and consumption, primarily by generating increasing scale economies in aggregate production. The resulting nonconvexities lead to multiple equilbria and hysteresis in some models.

The perceived problems with the neo-classical growth model are not inherent features of the model, but the consequences of assuming that the marginal product of capital diminishes to zero as the input of capital is increased indefinitely relative to labour. Instead of directly relaxing this assumption, the 'new' growth theorists in effect introduce a factor other than physical capital which is not subject to such inexorable diminishing returns. We take a different approach: we assume fertility and savings to be endogenous so that the rate of growth, labour and capital, and hence aggregate growth, to be endogenous. Second, we assume that population density has an external effect (not perceived by individual agents) on the production process either through negative congestion effect or through positive effect in stimulating innovation and technical change, so that the change in production possibilities is endogenous determined by fertility decisions of individual agents. Our model is not necessarily geared to generating balanced growth steady states and its non-linear dynamics generate a plethora of outcomes that include not only the steady state of the neo-classical model, but also growth paths not only without a steady state but are even chaotic. Per capita output grows exponentially (and super exponentially) in some of the examples.

KEY WORDS: Fertility, Technical Change, Growth

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1. Introduction

The consequences of private reproduction and capital (physical and human) accumulation decisions to long-run economic development have been the focus of research of a number of scholars in recent years (National Research Council (1986), Nerlove et al (1987), Raut (1985, 1991a, 1991b), and Simon (1977, 1981)). In the earlier literature on growth and development household formation, schooling, fertility and labour force participation decisions of households, their mortality experience and the resulting rate of population and labour force growth were assumed to be exogenous. The recent literature, in contrast, explicitly recognizes their endogeneity. In addition, greater emphasis is placed on human as contrasted with physical capital in the growth process.

The starting point of another strand of recent literature, labeled as "new" growth theory, is a misleading characterization of traditional neoclassical growth theory, namely, that it shows the steady state growth rate of income to be exogenous, and, in the absence of (exogenous) technical change (of a Harrod-Neutral type), this growth rate of income will equal the rate of growth of the labour force. Thus in the steady state per worker output and consumption are constant. A goal of 'new' theory is essentially to endogenize growth and to obtain sustained growth in per worker output and consumption, primarily by generating increasing scale economies in aggregate production. The resulting nonconvexities lead to multiple equilbria and hysteresis in some models.

It should be emphasized that per capita output can grow indefinitely even in traditional growth models if the marginal product of capital is bounded away from zero as the capital-labour ratio grows indefinitely. Thus the neoclassical assumption that the marginal product of capital is a strictly decreasing function of the capital-labour ratio is not inconsistent with indefinite growth of per capita output. It has to diminish to zero as the capital labour ratio increases indefinitely to preclude such growth.

Consider, for example, the simplest version of the neoclassical growth model (Solow (1956)). With a constant savings rate s, a constant rate of growth n of the labour force, no depreciation of capital and full employment the rate of growth $\dot{k}$ of the capital labour ratio k is given by

$$(1) \qquad \dot{k} = sf(k) - nk$$

where f(k) is average product of labour given constant returns to scale. It is straightforward to see that if $f(0) = 0$ and the marginal product of capital, i.e. $f'(k)$, is bounded away from n/s, that is, if $f'(k) > n/s$ for all k, then $\dot{k} > 0$ for all k. This in turn implies that the rate of growth of per worker output and consumption, namely $f'(k)\dot{k}/f(k)$, is positive. Moreover, given strict diminishing returns, i.e. $f''(k) < 0$, as $k \to \infty$, $f'(k)$ has a limiting value, say g, exceeding n/s. As such it can be verified that the asympotic growth rate of output and consumption will equal $sg - n > 0$. Since this is a function of s which is endogenous, growth is endogenous. To the extent the savings rate s is influenced by thriftiness of households, intertemporal preferences influence growth. However, it should be noted that if we assume that labour is essential to production, that is, output is zero if labour input is zero regardless of the level of capital input however large, then the limiting value of $f'(k)$ has to be necessarily zero. Thus for the limiting value to be positive a necessary condition is that labour is not essential to production.

Solow (1956) also showed that if the production function is such that the marginal product of capital increases up to some capital-labour ratio $\bar{k}$ and then decreases thereafter, multiple steady state equilibria are possible in each of which per capita output is constant but different across equilibria. Further, there is hysteresis in the sense that depending on the initial capital-labour ratio, per capita output will converge to different steady states. Based on this model, one can (and did!) make a case for a "big" push investment to escape from convergence to a stable low level equilibrium to a higher level equlibrium, a case that has been recently rediscovered and extended (Murphy et al (1989)). In these models, human capital plays no role.

Lucas (1988), on the other hand, postulates a production function to skill formation at the individual level that assumes that the rate of accumulation of skills is proportional to the level (or stock) of skills. Thus the marginal product of the stock of skills in terms of the rate of accumulation is a constant, given the time devoted to such accumulation. In addition he assumes that the average skill level of the entire labour force induces an externality to the production process, thus obtaining indefinite increasing returns to scale at the economy-wide level with respect to physical capital, labour force and its average skills. This naturally enables him not only to endogenize growth but also obtain a positive and sustained rate of growth of per capita output. Strictly speaking, even if the externality effect is absent, the assumption that the marginal product (in terms of rate of skill accumulation) of the stock of skills is constant rather than diminishing to zero is enough to obtain sustained growth. In Romer (1986) the stock of private knowledge at the level of the firm can be augmented through investment I in research through a constant returns to scale production function using I and k as inputs with bounded average product per unit of k. Thus there are strong diminishing returns to knowledge accumulation at the level of an individual firm. However, aggregate knowledge has increasing marginal productivity. Thus both Lucas and Romer in effect make assumptions that are analogous to the assumption in the Solow model that the marginal proudct of physical capital is bounded away from zero. Thus it is no surprise that both obtain sustained growth of per capita income. Both Lucas (1988) and Romer (1986) assume fertility to be exogenous. Ehrlich and Lui (1989) analyze a model in which human capital is the engine of growth and generate growth in per capita income and consumption as a result of accumulation of general and specific knowledge. They link longevity, fertility and economic growth through their interaction with human capital accumulation in an overlapping generations model with fertility as one of the endogenous choice variables.

It is clear that the two perceived problems with the neo-classical growth model, namely, that aggregate growth rate in the steady state is exogenous independent of intertemporal preferences and sustained growth in per capita income can come about only if there is (exogenous) technical progress are not inherent features of the model but the consequences of assuming that the marginal product of capital (or more generally of any reproducible factor) diminishes to zero as the input of capital (or that factor) is increased indefinitely relative to other inputs. Instead of directly relaxing this assumption about production technology, the 'new' growth theorists in effect introduce a factor other than physical capital (stock of skills in Lucas (1988), general knowledge in Romer (1986) etc) which is not subject to such inexorable diminishing returns. We take a different approach in this paper: first, by assuming fertility and savings to be endogenous, we make the growth in both inputs, labour and capital, and hence aggregate growth, to be endogenous in the absence of technical change. Second, by assuming that population density has an external effect (not perceived by individual agents) on the production process either through negative congestion effect or through positive effect in stimulating innovation and technical change, we make the change in production possibilities to be endogenous determined by fertility decisions of individual agents. However, unlike the "new" growth literature, our model, which is an extension of Raut (1985, 1991a), is not necessarily geared to generating balanced growth steady states. In fact, the non-linear dynamics of the model generates a plethora of outcomes (depending on the functional forms, parameters and initial conditions) that include not only the neo-classical steady state with exponential growth of population with constant per cpaita income and consumption, but also growth paths which do not converge to a steady state and are even chaotic. Per capita output grows exponentially (and super exponentially) in some of the examples.

2. Technological Change Induced by Population Density: A Model

E. Boserup (1989) and J. Simon (1981) among others have argued that the growth of population could itself induce technical change. In the Boserup model increasing population pressure on a fixed or very slowly growing supply of arable land induces changes in methods of cultivation, not simply through substitution of labour for land by choice of techniques within a known set of techniques but, more importantly, through the invention of new techniques. Simon also attributes a positive role for increases in population density in inducing technical progress. Neither of the two authors provides a complete theory of induced innovation. We do not provide one here either: we believe that the inducement to innovate will depend largely on the returns and risks to resources devoted to innovative activity and there is no particular reason to suggest that pre-existing relative factor prices or endowments will necessarily tilt these returns towards search of technologies that save particular factors. Instead, we simply analyze the implications of assuming that technical change is influenced by population density (strictly speaking, population size) in a world where fertility is endogenous.

More precisely, we assume that technical change in our model economy is Hicks-Neutral and its rate is determined by the change in the size of the working population. However, for both consumers and firms in this economy this is an externality. We introduce this externality in a model of overlapping generations in which a member of each generation lives for three periods, the first of which is spent as a child in the parent's household. The second period is spent as a young person working, having and raising children, as well as accumulating capital. The third and last period of life is spent as an old person in retirement living off support received from each of one's offspring and from the sale of accumulated capital. All members of each generation are identical in their preferences defined over their consumption in their working and retired periods. Thus, in this model the only reason that an individual would want to have a child is the support the child will provide during the parent's retired life. Production (of a single commodity which can be consumed or accumulated) is organized in firms which buy capital from the retired and hire the young as workers. Markets for product, labour and capital are assumed to be competitive.

Formally, a typical individual of the generation which is young in period t has n_t children (reproduction is by parthenogenesis!), consumes c_t^1, c_{t+1}^2 in periods t and t+1, and saves s_t in period t. She supplies one unit of labour for wage employment. Her income from wage labour while young in period t is w_t and that is the only income in that period. A proportion $\underline{a}$ of this wage income is given to parents as old age support. While old in period t+1, she sells her accumulated saving to firms and receives from each of her offspring the proportion $\underline{a}$ of his/her wage income. She enjoys a utility $u(c_t^1, c_{t+1}^2)$ from consumption. Thus her choice problem can be stated as:

Maximize $u(c_t^1, c_{t+1}^2)$ with respect to the non-negative variables c_t^1, c_{t+1}^2, s_t and n_t subject to:

$$(2.1) \qquad c_t^1 + \theta_t n_t + s_t \le (1-a)w_t$$

$$(2.2) \qquad c_{t+1}^2 \le q_{t+1} s_t + a w_{t+1} n_t$$

where θ_t is the output cost of rearing a child until young, q_{t+1} is the price of capital in period t+1 and w_t is the wage rate in period t where the numeraire in each period is that period's output.

It should be noted that restricting s_t to be non-negative implies that the young cannot borrow and spend more than their income (net of payment to their parents) to consume and spend on rearing children. This is a natural requirement since the only persons with resources to lend to the young are the old. But they will not lend since they will be dead when the loan is to be repaid. Of course, if there is a government, it can tax the old to transfer income to the young, but for the present let us assume that there is no government. Requiring n_t to be non-negative is also natural given that n_t are the number of offspring, though treating it as a continuous variable, while convenient, is not so natural! But leaving aside the absurdity of having a negative number of children, formally not allowing n_t to be negative is analogous to precluding borrowing. After all, borrowing is simply one way of increasing current consumption at the expense of future consumption. Letting n_t be negative will also increase current consumption at the expense of future consumption.

The firms of period t buy capital from the old at a price q_t per unit, pay wages at the rate of w_t per worker and maximize profits. Thus if they buy K_t units of capital and hire N_t workers, their profits π_t are given by

(2.3) $\pi_t = G(L_t)F(K_t,N_t) - q_tK_t - w_tN_t$

where $F(K_t,N_t)$ is a linear homogeneous function with strictly convex isoquants (which implies that F is concave) and $G(L_t)$ is the Hicks-Neutral productivity parameter that is assumed to depend on the number of young in period t. Of course if there is full employment N_t will equal L_t. Capital depreciates completely within one period.

Clearly the first order conditions for profit maximization (for t=0, 1,2...) are:

(2.4) $G(L_t)F_1(K_t,N_t) \leq q_t$ with equality if $K_t > 0$

(2.5) $G(L_t)F_2(K_t,N_t) \leq w_t$ with equality if $N_t > 0$

where F_i is the partial derivative of F with respect to its ith argument. It should be noted that in deriving (2.5) it is assumed that the producer chooses N_t without taking into account that under full employment N_t will equal L_t and hence affect $G(L_t)$. In other words, the possible effect of N_t on $G(L_t)$ is an uninternalized externality for the producer. If we assume that both inputs are essential to production so that $F(0,N) = F(K,0) = 0$ for all $N > 0$, $K > 0$, then positive production implies that (2.4) and (2.5) hold as equalities. Since F is homogeneous of degree one in K and N, F_i is homogeneous of degree zero in K and N. Thus F_i $(i = 1,2)$ is a function only of the ratio K_t/N_t. As such, the fact that (2.4) and (2.5) have to hold as equalities restricts the admissible set of q_t,w_t. Put another way, $q_t/G(L_t)$, $w_t/G(L_t)$ have to lie on the factor price frontier associated with F. Given an admissible pair (q_t, w_t) the profit maximizing value of $K_t/N_t = k_t$ is solved from

(2.6) $\quad F_1/F_2 = q_t/w_t$

Assume that F satisfies the Inada conditions, that is

$\lim_{k \to 0} \frac{F_1}{F_2} = \infty$ and $\lim_{k \to \infty} \frac{F_1}{F_2} = 0$. Then since $\frac{F_1}{F_2}$ is a decreasing function of k_t (because of strict convexity of isoquants of F), we obtain a unique solution for k_t as a function of q_t/w_t from (2.6). The strict convexity of isoquants of F also ensures that k_t is indeed a profit maximizing choice.

The first order conditions for consumer utility maximization are (for t = 0, 1, 2, ...):

(2.7) $\quad u_1(c_t^1, c_{t+1}^2) \leq \lambda_t$ $\quad$ with equality if $c_t^1 > 0$

(2.8) $\quad u_2(c_t^1, c_{t+1}^2) \leq \lambda_{t+1}$ $\quad$ with equality if $c_{t+1}^2 > 0$

(2.9) $\quad -\lambda_t \Theta_t + \lambda_{t+1}\, a\, w_{t+1} \leq 0$ $\quad$ with equality if $n_t > 0$

(2.10) $\quad -\lambda_t + \lambda_{t+1} q_{t+1} \leq 0$ $\quad$ with equality if $s_t > 0$

where u_i is the partial derivation of u with respect to its ith argument and λ_t is the lagrangean multiplier associated with the budget constraint in period t.

If we assume that u does not admit a satiation point, λ_t and λ_{t+1} will be positive and if both commodities are essential in consumption, then (2.7) and (2.8) will be equalities. For the model to be meaningful n_t and s_t have to be positive: if either is zero, since capital and labour are essential to production, the economy ceases to exist in period t+1! Thus

(2.9) and (2.10) have to be equalities as well which in turn means that

$$(2.11) \qquad q_{t+1} = \frac{\lambda_t}{\lambda_{t+1}} = \frac{a}{\theta_t} w_{t+1}$$

This is nothing but the arbitrage condition that the return from investing a unit of current income in physical capital, i.e. q_{t+1}, is the same as the return a/θ_t in investing it in and rearing $1/\theta_t$ children and obtaining a return of $\underline{a}\, w_{t+1}$ from each.

To proceed further with the analysis, one needs to specify the nature of dependence of θ_t on t. To the extent θ_t reflects the cost of parent's time, one may wish to make it a function of the ruling wage rate. Besides, one may also wish to incorporate the "environmental" effects of population density in child rearing costs. Setting θ_t equal to $\theta G(L_t) + \eta w_t$ will accomplish both. However, this formulation is much too general for analysis and only by restricting the form of utility and production functions it is possible to proceed further as in Sections 3b, 4a and 4b. However, if we set $\eta = 0$ and θ_t is a constant for all t, without such restrictions it is possible to obtain some results. We discuss this case in Section 3a.

3a. <u>Constant Child Rearing Costs: General Solution</u>

With θ_t a constant θ for all t, (2.11) uniquely determines q_{t+1}/w_{t+1} or equivalently $(q_{t+1}/G(L_{t+1}))/(w_{t+1})/G(L_{t+1}))$ as a function of a/θ for all t. This, together with the earlier result that $q_t/(G(L_t)$, $w_t/G(L_t)$ lie on the factor price frontier for F uniquely determine $q_t/G(L_t)$, $w_t/G(L_t)$ respectively as constants q^* and w^* for t = 1,2,... This means that q_t/w_t is also a constant q^*/w^* for t = 1,2... Given the capital stock k_o owned by the old and the number L_o of young at period zero (as determined a period earlier by the choice of the old living in period zero) and full employment of labour and capital, it is clear that q_o is $G(L_o)F_1(K_o,L_o)$ and w_o is $G(L_o)F_2(K_o,L_o)$. The capital-labour ratios from period 1 on are constant as determined by $F_1/F_2 = q^*/w^*$. Thus, with full employment ($L_t = N_t$), $k_t \equiv K_t/L_t$ is a constant k^* for $t = 1,2,\ldots$ But $K_t = L_{t-1}s_t$ and $L_t = L_{t-1}n_t$ so that $k^* = s_t/n_t$ for $t = 0,1,2\ldots$

Using (2.11) in constraints (2.1) and (2.2) one observes that $c_t^1 = (1-a)w_t - (\theta n_t+s_t)$ and $c_{t+1}^2 = (\theta n_t+s_t)\, q_{t+1}$. This means that θn_t+s_t ($\equiv z_t$) is the choice variable for utility maximization. The first order condition of utility maximization is $u_1/u_2 = q_{t+1}$. If we assume that consumption in both periods is normal and that the marginal utility of consumption in either period tends to ∞ if consumption in that period tends to zero, then u_1/u_2 is an increasing function of z_t rising from zero at $z_t = 0$ to ∞ as $z_t \to \infty$. Thus $u_1/u_2 = q_{t+1}$ can be solved uniquely for z_t as a function $H(w_t,q_{t+1})$ of w_t and q_{t+1} for $t = 0,1,2\ldots$ Since $s_t/n_t = k^*$, it follows that for $t = 0,1,2,\ldots$

(3.1) $$n_t = (\theta + k^*)^{-1}\, H(w_t,q_{t+1}) \text{ and } s_t = k^*(\theta+k^*)^{-1}\, H(w_t,q_{t+1})$$

Now $w_t = w^*G(L_t)$ $q_t = q^*G(L_t)$ for $t = 1,2\ldots$ As such if there is no technical change, i.e. $G(L_t)$ = constant for all t, the economy settles into a steady state $n_t \equiv n^* = (\theta + k^*)^{-1}\, H(w^*,q^*)$ and $s_t \equiv s^* - k^*(\theta + k^*)^{-1}\, H(w^*,q^*)$ from $t = 1$ on. This is the result reported in Raut (1991). Clearly with k_t constant, output per worker is constant from $t = 1$. Depending on whether $n^* \gtreqless 1$ the size of the economy increases, stays constant, or decreases to zero as $t \to \infty$.

3b. Constant Child Rearing Costs: Specific Examples

Suppose now that $G(L_t)$ is not a constant. To get some insight, let us assume that $u(c_t^1, c_{t+1}^2)$ is $(c_t^1)^\delta (c_{t+1}^2)^{1-\delta}$. Then $H(w_t, q_{t+1}) \equiv (1-\delta)(1-a)w_t$ so that

$$(3.2) \qquad n_t = \frac{L_{t+1}}{L_t} = \frac{(1-\delta)(1-a)}{(\Theta + k^*)} w^* G(L_t) \quad \text{or}$$

$$(3.3) \qquad L_{t+1} = \lambda L_t G(L_t) \quad \text{where}$$

$$(3.4) \qquad \lambda = \frac{(1-\delta)(1-a)w^*}{(\Theta + k^*)}$$

Clearly the behaviour of L_t will depend on the function $\lambda L_t G(L_t)$. If, for instance, $\lambda G(L_t) < 1$ for all $L_t > 0$, then L_t will decrease over time and converge to zero. Thus zero is the unique steady state value of L_t for such an economy. Let us assume away this uninteresting scenario of a declining economy and postulate that $\lambda G(L_t)$ exceeds 1 for L_t in some interval (a,b) where $a \geq 0$ and $b \leq \infty$. A plausible assumption is that $G(L_t)$ increases (i.e. there is increasing positive externality effect of population density) up to some $L_t = \bar{L}$ and then decreases to zero (i.e. there is an increasing negative externality effect of congestion) as L_t increases further. Alternatively, one could ignore congestion effects and assume that $G(L_t)$ is a logistic function with a positive asymptote. We explore both below. It is clear from (3.3) that a positive steady state value of L_t denoted by L^*, if it exists, is defined by $\lambda G(L^*) = 1$. (Of course $L = 0$ is always a steady state.) The following two examples illustrate some of the possibilities.

<u>Example 1</u> Let $G(L) = \gamma e^{-(L-\bar{L})^2/2}$ for $L \geq 0$. The curve $\lambda LG(L)$ reaches a unique maximum at $\bar{\bar{L}} = 0.5\ [\bar{L} + (\bar{L}^2 + 4)^{1/2}]$. Let $\bar{L} > 0$ and $\lambda\gamma > 1$. Then there are two possible steady state values L^* and L^{**} given by $L^* = \bar{L} - (2\mathrm{Log}\lambda\gamma)^{1/2}$ and $L^{**} = \bar{L} + (2\mathrm{Log}\ \lambda\gamma)^{1/2}$. $\lambda\gamma > 1$, it is clear that $L^{**} > \bar{L} > 0$ and $L^* > 0$ if $e^{\bar{L}^2/2} > \lambda\gamma$. Since $\bar{\bar{L}} > L^*$, the curve $\lambda LG(L)$ crosses the 45° from below so that the steady state L^* is unstable. If $\bar{\bar{L}} \geq L^{**}$ so that $\lambda LG(L)$ is non-decreasing while crossing the 45° line from above at L^{**}, L^{**} is locally stable so that L_t converges to L^{**} (zero) for any L_o above L^* (below L^*). Of course if $L_o = L^*$, L_t remains unchanged at L^* (Figure 1). On the other hand, if $\bar{\bar{L}} < L^{**}$ so that $\lambda LG(L)$ is decreasing while crossing the 45° line from above at L^{**}, there is a possibility of a limit cycle or even chaotic behaviour. In this example $L = 0$ is a locally stable steady state. In fact, for the following parameter values the Li-York sufficient condition for chaotic behaviour (see Baumol and Benhabib (1989)), namely, that there exist a value of L, say L_o, such that $L_o < L_1 < L_2$ and $L_3 < L_o$ is satisfied: $\lambda = 1.70$, $\gamma = 1$, $\bar{L} = 2.0$. Setting $L_o = 1.56$, one obtains $L_1 = 2.41$, $L_2 = 3.77$ and $L_3 = 1.34$. Chaotic behaviour of L_t can be seen in numerical simulations of this case (see Figures 2a and 2b). Convergence obtain for parameter values $\lambda = 1.2$, $\gamma = 1$, $\bar{L} = 0.7$. In this case L_t converges to 1.3. This is illustrated in Figures 3a and 3b.

Let us assume that the values of the parameters λ, γ, $\bar{L}$ are such that $\bar{\bar{L}} \geq L^{**}$ so that limit cycles and chaotic behaviour are ruled out. Then if L_t starts from any value larger than L^*, it converges to L^{**}, and the economy reaches a stationary state with a constant working population, constant capital labour ratio and hence constant output and wage per worker. It is to be noted that since $L^{**} > \bar{L}$ in the steady state there is congestion in the sense that $G(L)$ is decreasing at L^{**}. If the economy starts from any value less than L^* it converges to zero. Thus depending on the initial conditions the economy either converges to a 'good' stationary state L^{**} or collapses to zero!

Example 2

Let $G(L) = 1/(\alpha + \beta e^{-\gamma L})$ where $\alpha > 0$, $\beta > \lambda - \alpha > 0$, $\gamma > 0$
It is easily seen that $G'(L) > 0$ for all $L \geq 0$.

Clearly, there exists a unique steady state $L^* = 1/\gamma \operatorname{Log}(\beta/\lambda-\alpha) > 0$ at which $\lambda G(L^*) = 1$. Now $\phi(L) = \lambda L G(L)$ is an increasing function of L and $\phi'(L) = \lambda G(L) + \lambda L G'(L) > \lambda G(L) > \lambda G(L^*) = 1$ for $L > L^*$. Hence $\phi(L) - L$ is an increasing function of L for $L > L^*$. The steady state L^* is unstable. If the initial L is less than L^*, then L_t converges to zero and if it is greater than L^* it diverges to infinity (Figure 4). In the latter case, although the working population increases beyond limit, output per worker converges to $G(\infty)F(k^*,1) = (1/\alpha)F(k^*,1)$. The wage rate, price of capital, and consumption in each period of life of each generation also converge to constants. Thus an ever increasing population enjoys an unchanging standard of living.

The above examples show that as long as productivity as a function of the working population, that is $G(L_t)$, is bounded above, there is no possibility of sustained growth in output per worker in a laissez-faire competitive equilibrium. At best the economy may be able to support an ever increasing working population at a constant wage if $G(L_t)$ has a positive asymptote. At worst the economy will decline with the working population converging to zero asymptotically.

4.a Time Varying Child Rearing Costs, Cobb-Douglas Production and Utility Functions

Let us consider the case $\Theta_t = \Theta G(L_t) + \eta w_t$. It is clear that the arbitrage condition (2.11) continues to hold and, as such, the choice variable for utility maximization is still $z_t \equiv \Theta_t n_t + s_t$. As earlier, the first order condition of utility maximization can be solved to yield $z_t = H(w_t, q_{t+1})$. Using the arbitrage condition $q_{t+1}/w_{t+1} = a/\Theta_t$ in the first order conditions for profit maximization, one gets as before $F_1[K_{t+1}, L_{t+1}]/F_2[K_{t+1}, L_{t+1}] = a/\Theta_t$. Given our assumptions on F, this can be solved uniquely to yield $k_{t+1} \equiv K_{t+1}/L_{t+1} = h(a/\Theta_t)$, where h is a strictly decreasing function falling from ∞ as $a/\Theta_t \to 0$ to zero as $a/\Theta_t \to \infty$. Noting that $K_{t+1}/L_{t+1} = s_t/n_t$, we solve for n_t and s_t to obtain (for t=0, 1,...):

$$(4.1) \qquad n_t = \frac{H(w_t, q_{t+1})}{\Theta_t + h(a/\Theta_t)}, \qquad t \geq 0$$

$$(4.2) \qquad s_t = \frac{h(a/\Theta_t) H(w_t, q_{t+1})}{\Theta_t + h(a/\Theta_t)}$$

$$(4.3) \qquad w_o = G(L_o)[f(k_o) - k_o f'(k_o)] \quad \text{where } k_o = K_o/L_o.$$

$$(4.4) \qquad w_{t+1} = G(L_{t+1}) \left[f[h(a/\Theta_t)] - h(a/\Theta_t)\, f'[h(a/\Theta_t)]\right]$$

$$(4.5) \qquad q_{t+1} = G(L_{t+1}) f'[h(a/\Theta_t)] \qquad t \geq 0$$

$$(4.6) \qquad L_{t+1} = L_t \; n_t$$

$$(4.7) \qquad \Theta_t = \Theta G(L_t) + \eta\, w_t$$

It is not easy to derive even the qualitative properties of the solution to the above set of difference equations. Once again to gain some insights let us assume as before that the utility function is $(c_t^1)^\delta (c_{t+1}^2)^{1-\delta}$ and further the production function $F(K,L)$ is $K^{\sigma}L^{1-\sigma}$. Then, $h(a/\Theta_t) = (\sigma/1-\sigma)(a/\Theta_t)^{-1}$ and $H(w_t, q_{t+1}) = (1-\delta)(1-a)w_t$. The system of equations (4.1)-(4.7) can be reduced to two basic difference equations.

$$(4.8) \quad k_{t+1} = \frac{\sigma}{1-\sigma} \cdot \frac{\Theta_t}{a} = \frac{\sigma}{1-\sigma} \cdot \frac{1}{a} \cdot [\Theta G(L_t) + \eta\, w_t]$$

$$= \frac{\sigma}{1-\sigma}\ \frac{1}{a}\ G(L_t)\ [\Theta + \eta(1-\sigma)k_t^{\sigma}]$$

$$(4.9) \quad \frac{L_{t+1}}{L_t} \equiv n_t = \frac{(1-\delta)a(1-a)(1-\sigma)^2 k_t^{\sigma}}{[a(1-\sigma)+\sigma][\Theta+\eta(1-\sigma)k_t^{\sigma}]}$$

$$= \left[\frac{(1-a)\ (1-\delta)}{\eta}\right] \left[\frac{a(1-\sigma)}{a(1-\sigma) + \sigma}\right] \left[\frac{\eta(1-\sigma)k_t^{\sigma}}{\Theta+\eta(1-\sigma)k_t^{\sigma}}\right]$$

Now $n_t < \frac{(1-a)(1-\delta)}{\eta}$. As such, if $\frac{(1-a)(1-\delta)}{\eta} < 1$ it is seen from (4.9) that the working population converges to zero as $t \to \infty$ regardless of the process $G(L_t)$. This holds even if $G(L_t)$ is purely time dependent as for example $G(L_t) = (1+\epsilon)^t$ with $\epsilon > 0$ so that total factor productivity grows exponentially in time! However the welfare $u(c_t^1, c_{t+1}^2)$ of each member of the declining working population increases over time.

4b. Cost of Child Rearing Proportional to the Wage Rate: Some Examples

Two special cases are of some interest. Suppose $\Theta = 0$ so that the cost of rearing a child at time t is proportional to the wage rate at time t. Then we see from (4.9) that n_t, the growth in working population, is a constant

$n^* = \frac{a(1-a)(1-\sigma)(1-\alpha)}{\eta[a(1-\sigma)+\sigma]}$ (independent of the process $G(L_t)$) so that

$L_t = L_o(n^*)^t$. From (4.8) we note that

(4.10) $\text{Log } k_{t+1} = \text{Log } G\,[L_t] + \text{Log } \sigma + \text{Log } \eta - \text{Log } a + \sigma \text{ Log } k_t$

Denoting Log k_t by x_t, Log $G(L_t)$ by g_t and Log σ + Log η - Log a by ω, the solution to (4.10) is

(4.11) $$x_{t+1} = \frac{\omega(1-\sigma^{t+1})}{(1-\sigma)} + x_o\sigma^t + \sum_{\tau=0}^{t} \sigma^\tau g_{t-\tau}$$

If as in Example 2, $G(L) = [\alpha + \beta e^{-\gamma L}]^{-1}$, $a > 0$, $b > 0$, $\gamma > 0$, then G(L) is bounded and converges to $1/\alpha$ or $1/(\alpha + \beta)$ depending as whether $L_t \to \infty$ (i.e. $n^* > 1$) or $L_t \to 0$ (i.e. $n^* < 1$) as $t \to \infty$. Hence using (4.11) and noting that $0 < \sigma < 1$, we can say that as $t \to \infty$, x_t converges in either case. The average and marginal product of labour, and hence the welfare of each member of a generation, also converge to constants, with the working population increasing indefinitely in the first case and dwindling to zero in the second. More generally, if $G(L) > 0$ is bounded above, then (4.11) implies that $(1-\sigma)x_t$ is bounded above as well, so that the welfare of each member of any generation is bounded above, with the size of the working population growing or dwindling depending on whether n^* is greater or less than 1.

What if $n^* > 1$ (so that $L_t \to \infty$ as $t \to \infty$) and G(L) is unbounded? Suppose $G(L_t)$ behaves (for large values of L_t) as $e^{\mu L_t}$ ($\mu > 0$) so that g_t behaves as $\mu L_t = \mu L_o(n^*)^t$. Then from (4.11) it follows that x_{t+1} (for large values of t) is

(4.12) $$\frac{\omega(1-\sigma^{t+1})}{1-\sigma} + x_o\sigma^t + \mu L_o(n^*)^t \frac{[1 - (\sigma/n^*)^{t+1}]}{1 - \sigma/n^*}$$

Since $n^* > 1 > \sigma$, as $t \to \infty$, x_{t+1} behaves as $\mu L_o(n^*)^t/(1-\sigma/n^*) + \omega/1-\sigma$. This in turn means that k_t behaves asymptotically as $\exp(\mu L_o(n^*)^t/1 - \sigma/n^*)$ and the average product of labour $= G(L_t)k_t^\sigma$ behaves as $e^{(\sigma + \upsilon L_o)(n^*)^t}$ where υ is a positive constant! Thus one obtains super-exponential growth. On the other hand, if $G(L_t)$ behaves like $A(L_t)^\mu$ for large values of L_t ($\mu > 0$ implies that

$G(L_t)$ is still unbounded) then $g(L_t)$ behaves like μ log $L_t = \mu[\log L_o + tn^*]$. From (4.11) it can be shown that x_{t+1} behaves as $\mu n^* t/1-\sigma$ large values of t. This means that k_t grows exponentially at the rate $\mu n^*/1-\sigma$. With $G(L_t)$ behaving like $[L_o(n^*)^t]^\mu = (L_o)^\mu (n^*)^{\mu t}$, exponential growth of k_t implies exponential growth in the average and marginal product of labour and in the welfare of each member of a generation.

4.c Cost of Child Rearing Proportional to Externality Effect: Some Examples

The second special case is $\eta = 0$. This implies that the child-rearing cost is proportional to the externality factor $G(L_t)$ but does not depend on the wage rate. This is not an altogether implausible case if the factors that bring about positive (or negative) externalities associated with population density (e.g. congestion or economies of scale in schooling) also influence the cost of child rearing. The assumption that $\Theta_t = \Theta G(L_t)$ is a simple representation of this effect. Retaining the assumptions that both the utility and production functions are Cobb-Douglas, we get the basic equations:

(4.13) $$\frac{L_{t+1}}{L_t} = n_t = \frac{(1-\delta)a(1-a)(1-\sigma)^2}{[a(1-\sigma)+\sigma]\Theta} k_t^\sigma$$ and

(4.14) $$k_{t+1} = \frac{\sigma}{1-\sigma} \frac{\Theta}{a} G(L_t)$$ or

(4.15) $$L_{t+1} = \frac{(1-\delta)a(1-a)(1-\sigma)^2}{[a(1-\sigma)+\sigma]\Theta} \left[\frac{\sigma}{1-\sigma} \frac{\Theta}{a} G(L_{t-1}) \right]^\sigma L_t$$

Consider the case where $G(L_t) = AL_t^\mu$. Substituting in (4.15), taking logarithms of both sides, defining $\ell_{t+1} \equiv \text{Log } L_{t+1}$ and

$$\bar{w} \equiv \text{Log} \left[\frac{(1-\delta)a(1-a)(1-\sigma)^2}{[a(1-\sigma)+\sigma]\Theta} \left(\frac{\sigma A \Theta}{(1-\sigma)a} \right) \right]$$ we get:

(4.16) $$\ell_{t+1} - \ell_t - \mu\sigma \ell_{t-1} = \bar{w}.$$ The solution of (4.16) is

(4.17) $\ell_t = -\frac{\bar{w}}{\mu\sigma} + D_1\rho_1^t + D_2\,\rho_2^t$

where $\rho_1 = \frac{1 + (1+4\mu\sigma)^{1/2}}{2}$ $\qquad \rho_2 = \frac{1 - (1+4\mu\sigma)^{1/2}}{2}$

The initial conditions $D_1 + D_2 - \bar{w}/\mu\sigma = \text{Log } L_o$ and $D_1\rho_1 + D_2\rho_2 - \bar{w}/\mu\sigma =$ $\text{Log } L_o + \text{Log } n_o$ determine D_1 and D_2. Of course n_o depends on the given k_o. Solving for D_1 and D_2, we get:

(4.18) $D_1 = \left[\left(\frac{\bar{w}}{\mu\sigma} + \text{Log } L_o \right) (1-\rho_2) + \text{Log } n_o \right] (1 + 4\mu\sigma)^{-1/2}$

(4.19) $D_2 = \left[\left(\frac{\bar{w}}{\mu\sigma} + \text{Log } L_o \right) (\rho_1 - 1) - \text{Log } n_o \right] (1 + 4\mu\sigma)^{-1/2}$

For small values of $\mu\sigma$, $(1 + 4\mu\sigma)^{1/2} \simeq 1 + 2\mu\sigma$, so that we see from (4.17) that $\ell_t = \text{Log } L_t$ grows asymptotically at the rate $\mu\sigma$. From (4.15), it follows that Log k_{t+1} also grows at the same rate as well. Hence k_t and L_t grow super-exponentially.

Consider the initial values of L_o and k_o given by:

(4.20) $\frac{(1-\delta)a(1-a)(1-\sigma)^2}{[a(1-\sigma)+\sigma]\Theta} k_o^{\sigma} = 1$ and

(4.21) $\frac{\sigma}{1-\sigma} \frac{\Theta}{a} G(L_o) = k_o$

It is clear then from repeated application of (4.14) and (4.15) that k_t and L_t remain at L_o and k_o so that these are steady state values. Further, any values other than these will lead to either $L_1 \neq L_o$ or $k_1 \neq k_o$ or both so that the economy will not be in a steady state. If we assume that (4.20) and (4.21) have unique solutions, then the steady state of the model is unique.

It is easily verified that if $G(L) = AL^{\mu}$, equations (4.20) and (4.21) imply $\bar{w} = -\mu\sigma$ Log L_o and $n_o = 1$ so that from (4.18) and (4.19) we find $D_1 = D_2 = 0$ so that the economy remains in a steady state from period zero. The fact that other values of k_o and L_o can lead to super growth in this special case of $G(L) = AL^{\mu}$ suggests that in the general model convergence to the unique steady state is not assured. It is also clear from (4.15) that if G(L) is bounded, k_t is bounded and if the right-hand side of (4.14) evaluated at the upper bound (lower bound) of k_t is less (greater) than unity, L_t declines to zero (increases beyond limit) over time. However, with k_t and $G(L_t)$ bounded, the average and marginal productivity of labour are bounded even if L_t is unbounded. Thus in the case of $\eta = 0$, as in the case of $\Theta = 0$, we can generate an economy with super exponential growth, an economy which eventually disappears altogether or an economy with ever increasing labour force enjoying bounded levels of living for appropriate choice of the functional form G(L) and the initial conditions.

5. Social Planner's Optimum and Public Policy Intervention to Sustain It

The discussion so far looked at laissez-faire competitive equilibrium paths. Since the framework involves an externality that is not internationalized by any of the agents, it is worth examining the implications of a social planner internalizing it and whether the social planner's optimum can be realized as a private optimum given suitable public policy interventions in the form of taxes and subsidies.

For this purpose let us assume that the social planner maximizes the discounted sum (with a discount factor β, $0 < \beta < 1$) of the utility of a member of each generation multipled by the number of individuals in that generation. Since individuals consume only when they are working and when they are old, generations are indexed by the period t when they are working. In period 1, the number of persons in the last period of life (i.e. the number of individuals of generation zero) and the number of working young (i.e. the number of individuals of generation 1 which is also equal the total number of children that members of generation zero had in period zero) are both prodetermined $\bar{L}_o$ and $\bar{L}_1$, respectively. The planner can choose the number of members of all subsequent generations, i.e. L_t for $t \geq 2$. The consumption of a member of generation zero in period zero also predetermined at $\bar{c}_0^1$. But their consumption c_1^2 in the last period of their life, namely, in period 1, (and hence their utility $u(\bar{c}_0^1, c_1^2)$ and the consumption while working and while old (and hence their utility) of all other generations, $\{c_t^1, c_{t+1}^2\}$ $(t = 1,2,\ldots)$, are subject to choice by the planner. The savings of a member of generation 0 while working is predetermined at $\bar{s}_o$ while the savings of a member of every other generation, s_t $(j = 1,2,\ldots)$, are again subject to choice. For simplicity the cost of child-rearing is assumed to be θ per child at all t. Thus the planner's problem then is to:

$$\text{Maximize} \sum_{t=0}^{\infty} L_t \beta^t u(c_t^1, c_{t+1}^2) \quad \text{subject to}$$

(5.1) $$L_{t-1}c_t^2 + L_t(c_t^1 + s_t) + L_{t+1}\theta \leq G(L_t)F(L_{t-1}s_{t-1}, L_t), \quad t = 1,2,\ldots$$

(5.2) $$L_0 = \bar{L}_0, \; L_1 = \bar{L}_1, \; c_0^1 = \bar{c}_0^1, \; s_0 = \bar{s}_0$$

(5.3) $$L_t \geq 0, \; c_t^i \geq 0 \quad i = 1,2, \quad t = 0,1,2\ldots$$

Assuming a solution exists and it is an interior one, the first order conditions for t = 1, 2, ... are seen to be as follows (where ϵ_t is the shadow price of constraint (5.1) at time t, $u^t \equiv u(c_t^1, c_{t+1}^2)$, $F^t \equiv F(L_{t-1}s_{t-1}, L_t)$ and subscript j of a function denotes the partial derivative with respect to its jth argument):

	Choice Variable	First Order Condition
(5.4)	c_t^1	$L_t \beta^t u_1^t = \epsilon_t L_t$
(5.5)	c_t^2	$L_{t-1} \beta^{t-1} u_2^{t-1} = \epsilon_t L_{t-1}$
(5.6)	s_t	$\epsilon_t L_t = \epsilon_{t+1} L_t G(L_{t+1}) F_1^{t+1}$
(5.7)	L_{t+1}	$\epsilon_t \Theta + \epsilon_{t+1}(c_{t+1}^1 + s_{t+1}) + \epsilon_{t+2} c_{t+2}^2 =$ $\beta^{t+1} u^{t+1} +$ $\epsilon_{t+1} \quad [G_1(L_{t+1}) F^{t+1} + G(L_{t+1}) F_2^{t+1}] +$ $\epsilon_{t+2} G(L_{t+2}) F_1^{t+2} \; s_{t+1}$

Now simplifying, (5.4) - (5.6) become

(5.4)' $\beta^t u_1^t = \epsilon_t$

(5.5)' $\beta^{t-1} u_2^{t-1} = \epsilon_t$

(5.6)' $\epsilon_t = \epsilon_{t+1} G(L_{t+1}) \; F_1^{t+1}$.

Substituting then in (5.7), it is seen that

$$(5.7)' \quad \epsilon_t \Theta = \beta^{t+1}(u^{t+1} - u_1^{t+1} \; c_{t+1}^1 - u_2^{t+1} \; c_{t+1}^2) + \epsilon_{t+1} \; [G_1(L_{t+1}) \; F^{t+1} + G(L_{t+1}) F_2^{t+1}]$$

Now (5.4)′ - (5.7)′ together imply that

(5.8) $$\frac{\epsilon_t}{\epsilon_{t+1}} = \frac{u_1^t}{u_2^t} = G\,(L_{t+1})\,F_1^{t+1} \quad \text{and}$$

(5.9) $$\frac{\epsilon_t}{\epsilon_{+1}}\,\theta = \frac{\beta^{t+1}}{\epsilon_{t+1}}\left(u^{t+1} - u_1^{t+1}\,c_{t+1}^1 - u_2^{t+1}\,c_{t+1}^2\right)$$

$$+ G(L_{t+1})\,F_2^{t+1}\left[1 + \frac{G_1\left(L_{t+1}\right)\,F^{t+1}}{G(L_{t+1})F_2^{t+1}}\right]$$

On the other hand, from the first-order conditions of private (consumer and producer) optimization (equations (2.4) - (2.11)) it is seen that

(5.10) $$\frac{\lambda_t}{\lambda_{t+1}} = \frac{u_1^t}{u_t^2} = q_{t+1} = G(L_{t+1})\,F_1^{t+1} \quad \text{and}$$

(5.11) $$\frac{\lambda_t}{\lambda_{t+1}}\,\Theta = a\,G(L_{t+1})\,F_2^{t+1}$$

A comparison of (5.8) with (5.10) shows that if $\frac{\epsilon_t}{\epsilon_{t+1}} = \frac{\lambda_t}{\lambda_{t+1}}$,

<u>given the same values</u> for L_t and L_{t+1}, the private and socially optimal consumption and savings decisions would be the same. Now $\epsilon_t/\epsilon_{t+1}$ is the value of a unit of output available in period t+1 in units of output of period t, i.e. it is the social discount factor for output, and λ_t/λ_{t+1} is the corresponding discount factor in the private market equilibrium. However, as is to be expected, a comparison of (5.9) and (5.11) shows that even if there were no externalities associated with population, i.e. $G_1(L_{t+1}) = 0$ and that $\epsilon_t/\epsilon_{t+1} = \lambda_t/\lambda_{t+1}$, private and social decisions with respect to fertility will in general differ. Even though the <u>cost</u> of an additional child, i.e. the cost of child rearing, is the same for private and social

decisions, under the assumption $\epsilon_t/\epsilon_{t+1} = \lambda_t/\lambda_{t+1}$, the benefits represented by the right-hand sides differ.

A selfish parent in her private decision counts as benefit only the proportion <u>a</u> of the wage $G(L_{t+1})F_2^{t+1}$ earned by the additional child. However, for the society there are three contributions of an extra child, represented by three terms of the right-hand side of (5.9). The term$(\beta^{t+1}/\epsilon_{t+1})$ $(u^{t+1} - u_1^{t+1} c_{t+1}^1 - u_2^{t+1} c_{t+2}^2)$ represents (in units of present value of output of period t+1) the contribution to welfare of an extra person in period t+1, i.e. her utility u^{t+1} net of the cost of her lifetime consumption, i.e. $u_1^{t+1} c_{t+1}^1 + u_2^{t+1} c_{t+2}^2$ and for a concave utility function this is non-negative (being zero for a linear homogeneous utility function). This term arises from the fact that social welfare is utilitarian with respect to each generation in that it adds up the utility of all members of the utility function in determining the contribution of each generation to welfare. Thus this contribution is $L_t u^t$. If instead one considered the contribution of each generation to be the welfare of a representative agent, it would be only u^t. In the latter case, the first order conditions (5.4)'-(5.6)' will remain unchanged and in (5.7)' (and hence in 5.9) the first term will be absent. Whether one should take a utilitarian point of view is, largely though not entirely, an ethical issue (Koopmans (1967) and Nerlove et al (1987)) which we do not wish to pursue here. The term $G(L_{t+1})F_2^{t+1}$ is the contribution to output of an extra person in period t+1 and the last term $G_1(L_{t+1})F^{t+1}$ is the externality effect of that extra person.

It is evident from the above comparison if the planner with perfect foresight can set taxes or subsidies on income transferred by a child to her parent so as to reflect social considerations, thus making the net of tax or subsidy contribution equal to the social value of an extra child, social optimum can be realized as a private market equilibrium. The required net tax (τ) (in fact, it is a subsidy since it is negative) is

$$(5.12) \quad \tau = a - \frac{\beta^{t+1}}{\epsilon_{t+1} G(L_{t+1}) F_2^{t+1}} \left[u^{t+1} - u_1^{t+1} c_{t+1}^1 - u_2^{t+1} c_{t+2}^2 \right] - 1 - \frac{G_1(L_{t+1}) F^{t+1}}{G(L_{t+1}) F_2^{t+1}}$$

Thus, each parent in deciding how many children to have will take into account that each child will provide a proportion a of her wages to the parent and the government will provide a proportion $-\tau$ of each child's wage. In all, the parent, in her old age, received a proportion $a - \tau$ (recall $\tau < 0$) of each of her children's wage. By definition, the cost of subsidy will be financed by a lump sum tax. From (5.12) it is seen that if there is no externality ($G_1 = 0$) and if u is linear homogeneous $\tau = a - 1$ or $a - \tau = 1$. This means that fertility decisions would be made in the expectation that the entire wage of each child will accrue to the parent in her old age!

6. Conclusions

To conclude, with endogenous fertility and endogenous technical change arising from externalities associated with labour force growth, the problem of stagnant steady state per worker consumption (as in the neo-classical growth model with exogenous fertility and no technical change) is not necessarily avoided. Whether in fact a steady state exists, whether it is unique and stable and whether per capita consumption grows indefinitely all depend on preferences, technology and the nature of externality associated with labour force growth. Only sound empirical research will shed light on this issue.

Private fertility decisions (under selfish preferences) may be non-optimal from a social perspective even in the absence of externalities associated with population growth. Externality adds yet another reason for this divergence. However, as is to be expected, a planner with perfect foresight can realize the social optimum through private decisions by appropriately taxing or subsidizing the intergenerational transfer that is distorted, namely, the payment made by the working young to their parent.

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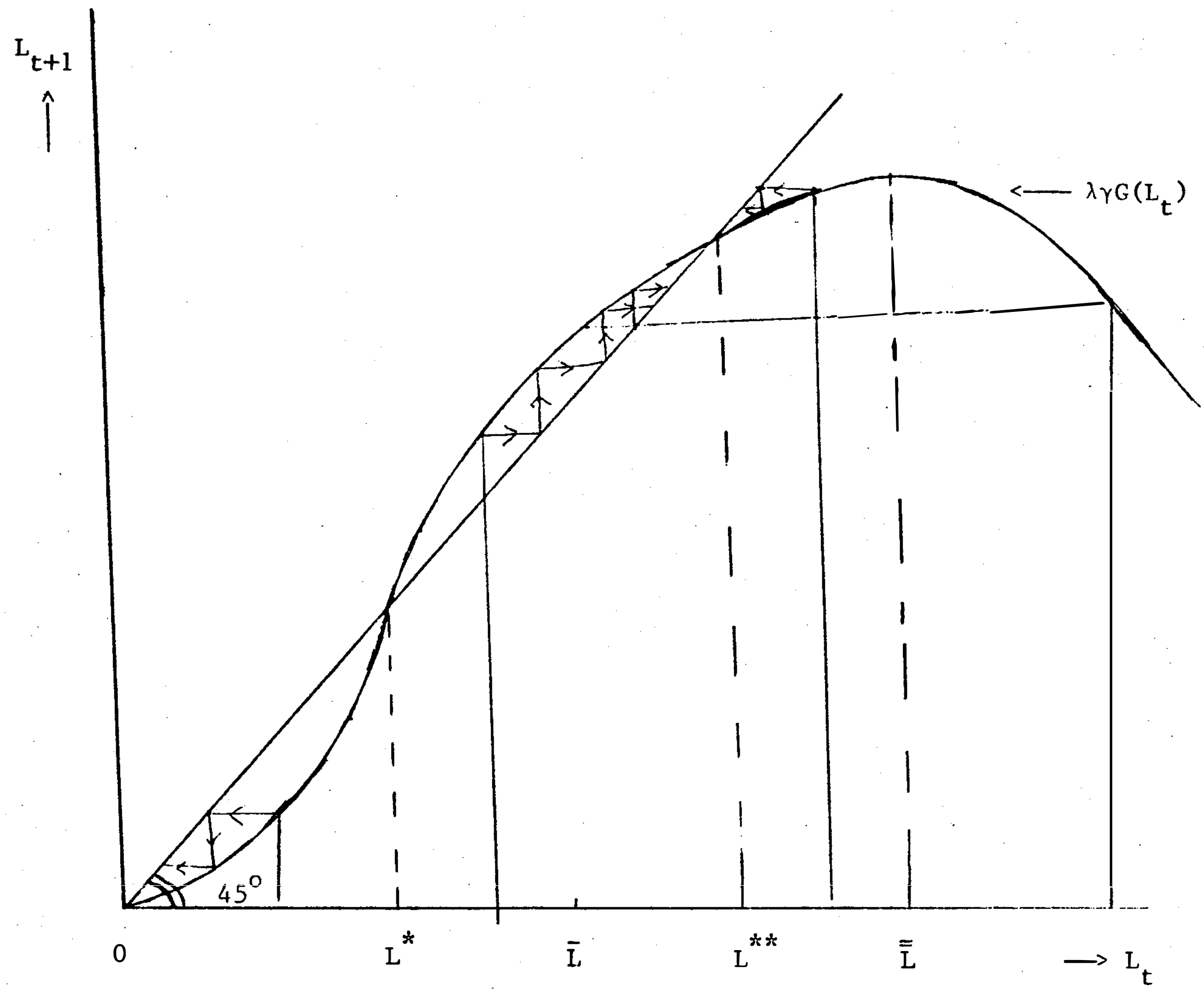


Figure 1

Figure 2a

Parameters $\lambda = 1.70$, $\gamma = 1.0$, $\bar{L} = 2.0$

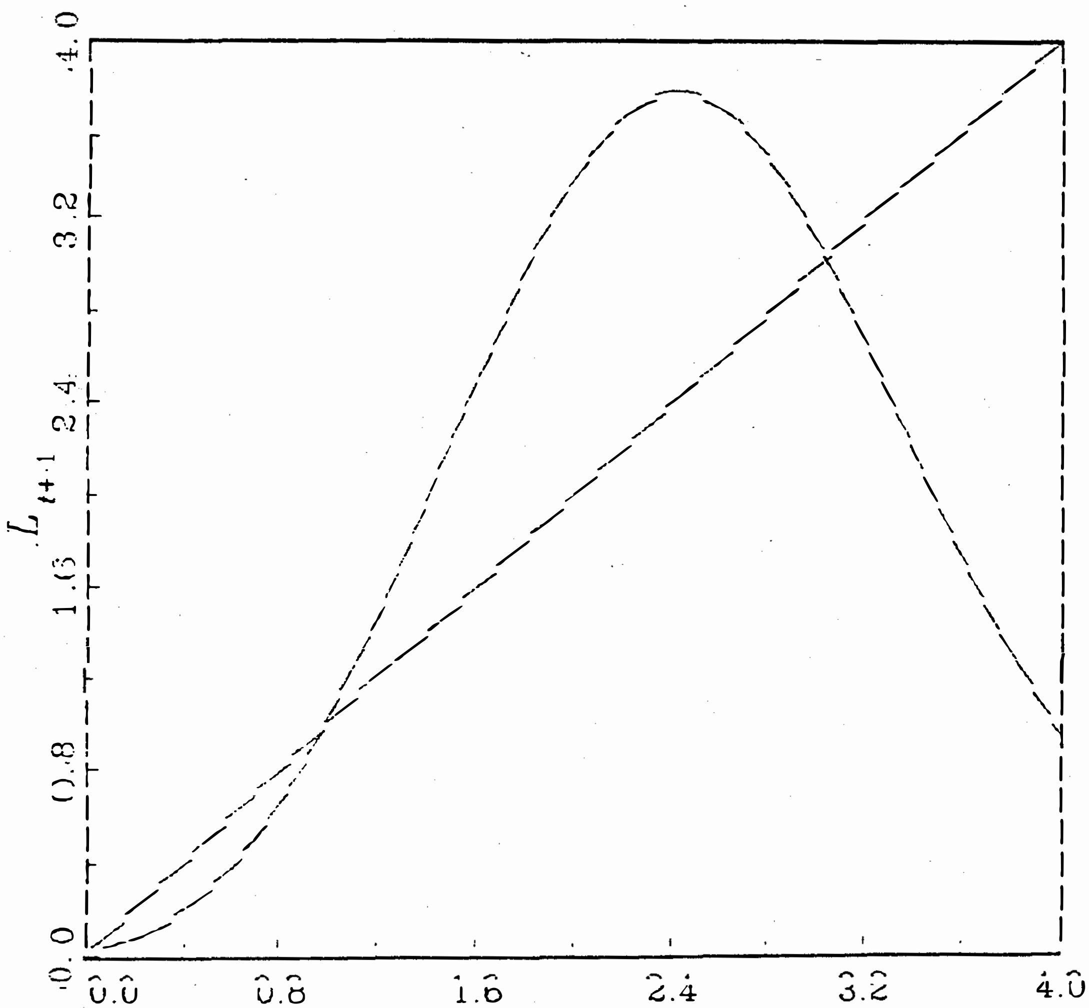
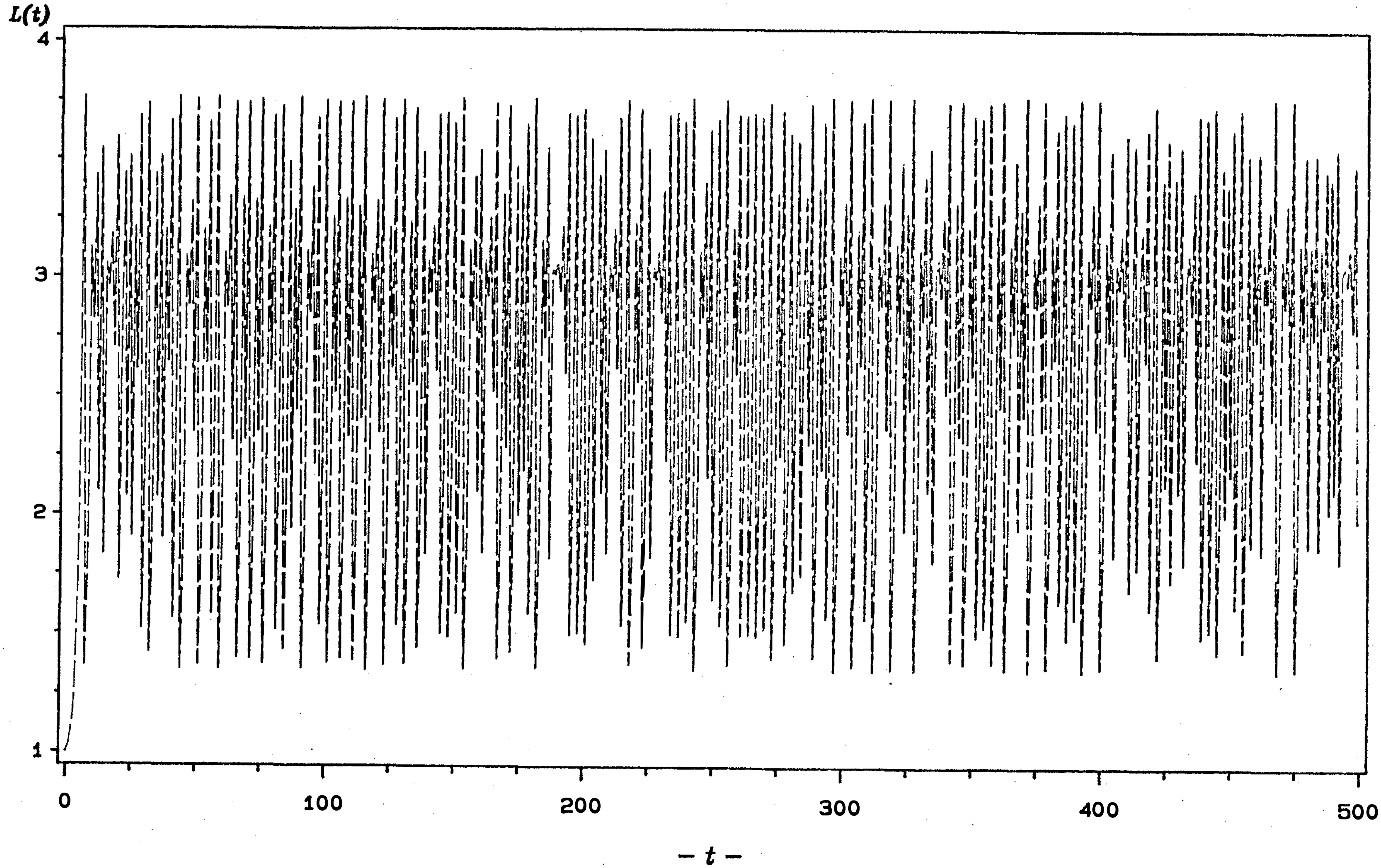


Figure 2b

Parameters $\lambda = 1.70$, $\gamma = 1.0$, $\bar{L} = 20$



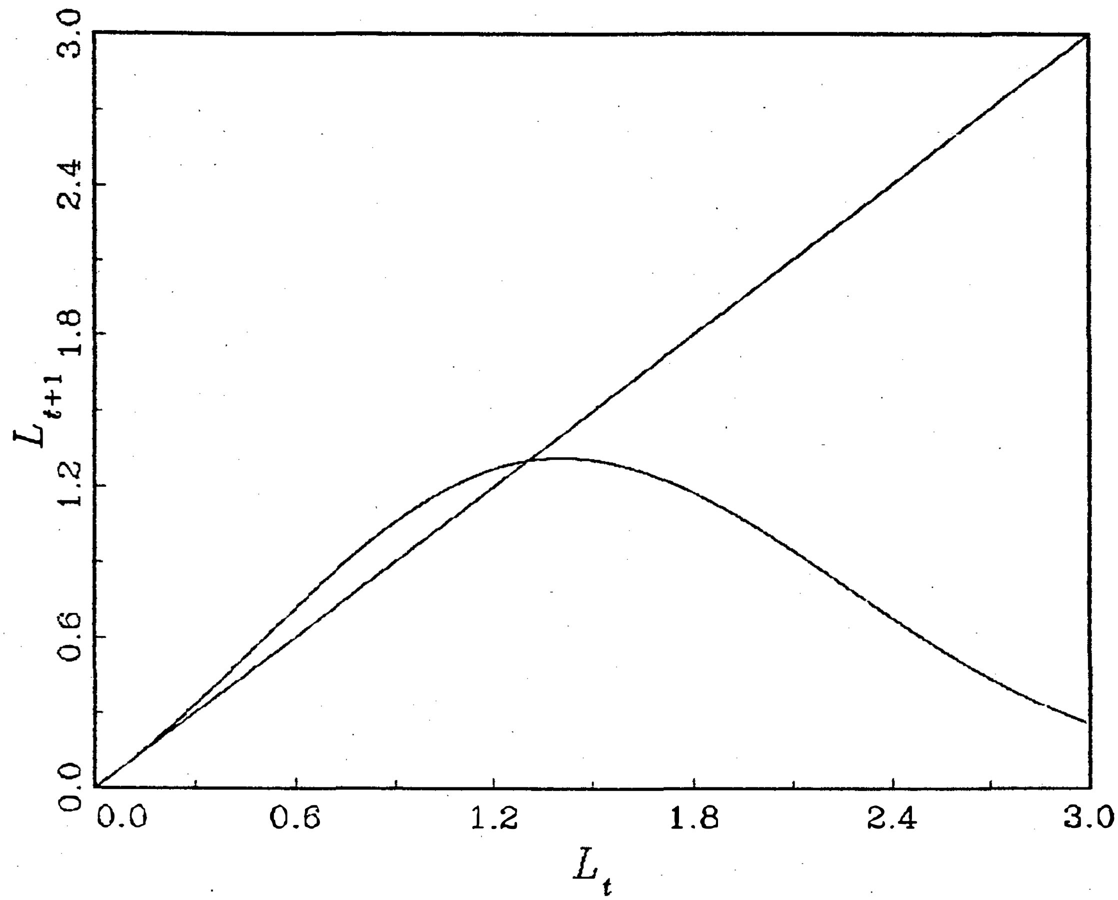
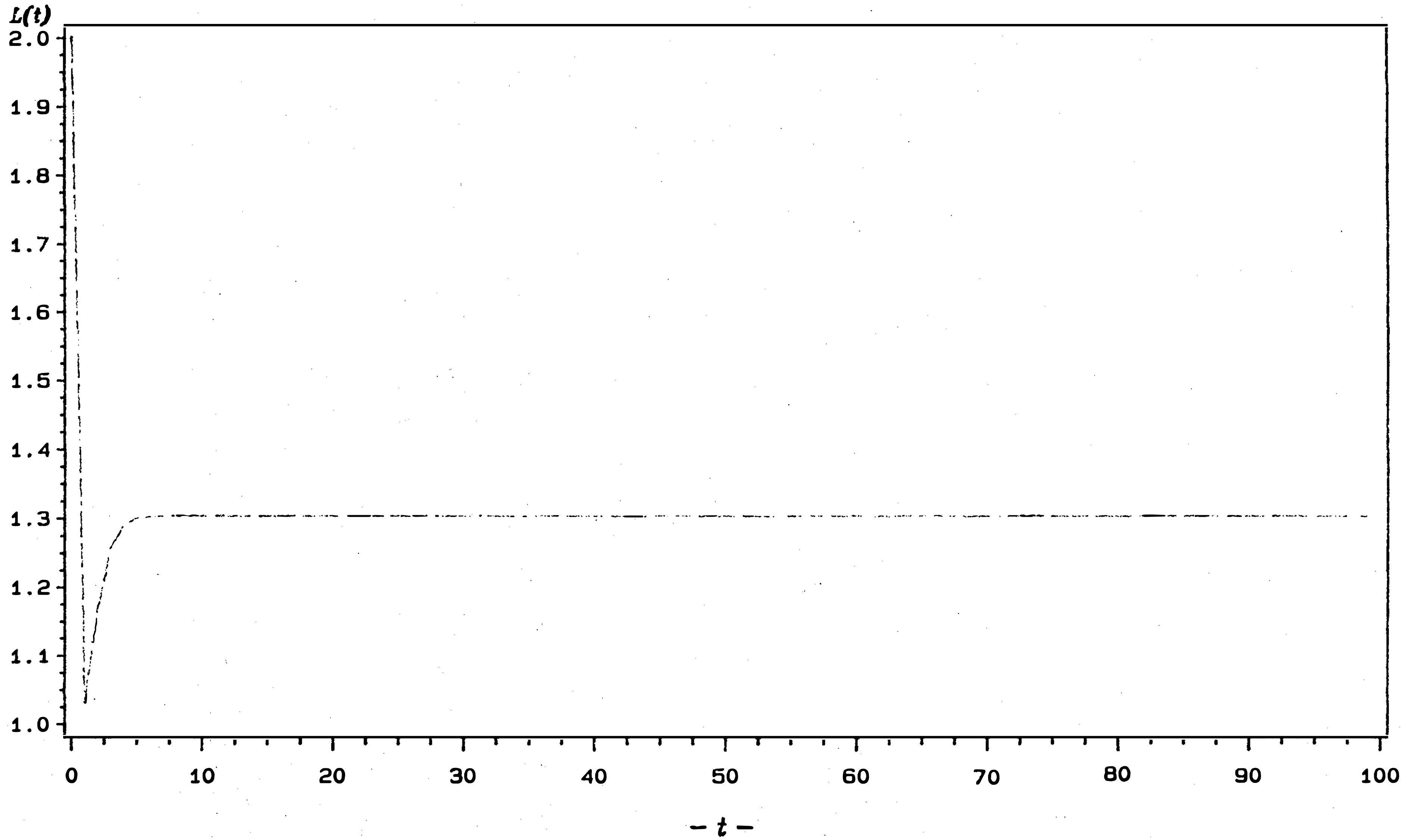


Figure 3a

Parameters λ = 1.2, γ = 1.0, $\bar{L}$ = 0.7

Figure 3b

Parameters $\lambda = 1.2$, $\gamma = 1.0$, $\bar{L} = 0.7$



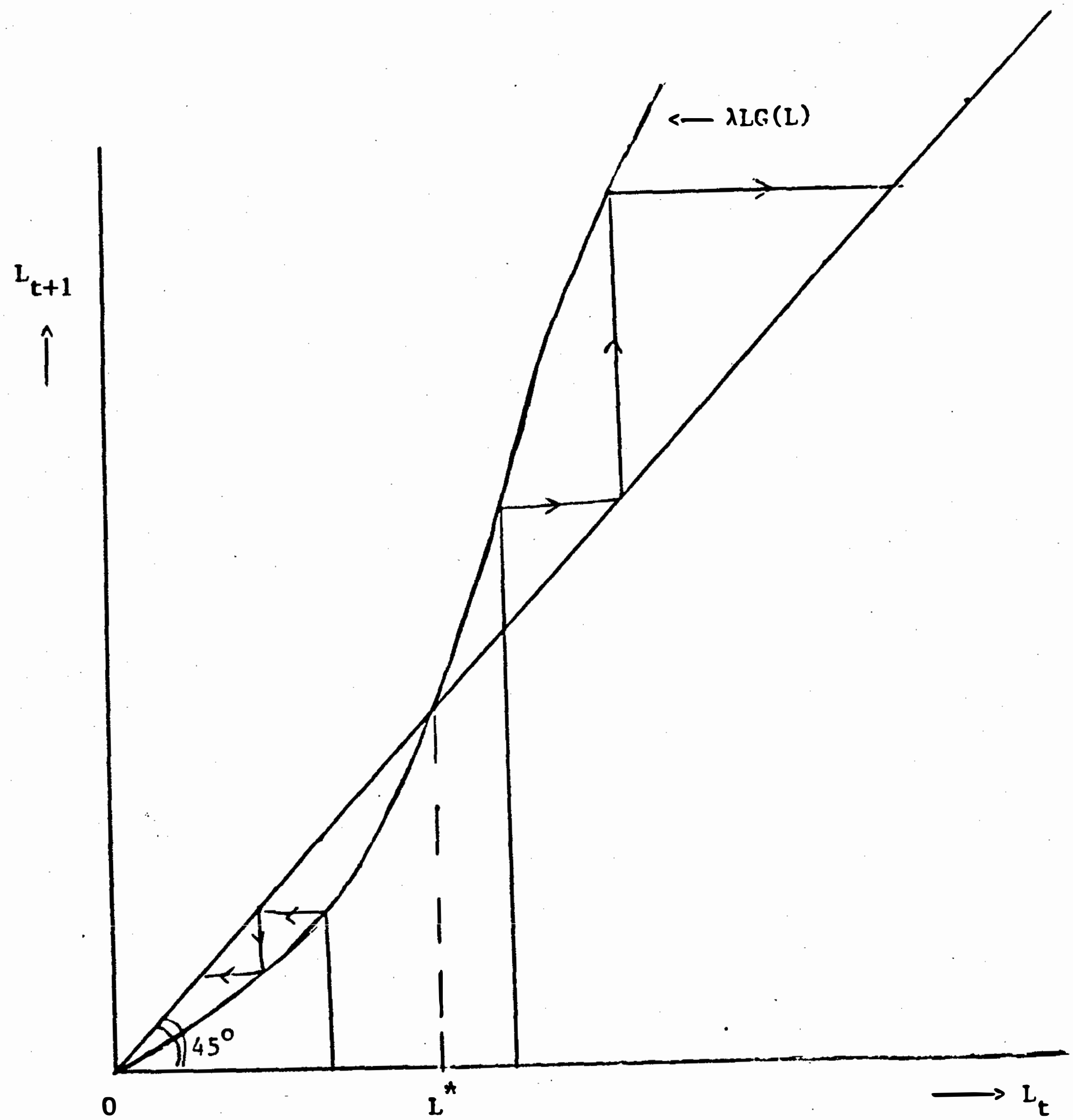


Figure 4