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**A DURATION ANALYSIS OF FERTILITY  
BEHAVIOR OF MALAYSIAN WOMEN:  
PARTIAL LIKELIHOOD AND  
NONPARAMETRIC MAXIMUM  
LIKELIHOOD ESTIMATES**

by

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## ABSTRACT

### A DURATION ANALYSIS OF FERTILITY BEHAVIOR OF MALAYSIAN WOMEN: PARTIAL LIKELIHOOD AND NONPARAMETRIC MAXIMUM LIKELIHOOD ESTIMATES

In this paper, we estimate a multi-spell duration model of timing of marriage and timing and spacing of children by the Malaysian women. We compute the Heckman-Singer maximum likelihood estimates that control for unobserved heterogeneity nonparametrically, maximum likelihood estimates without correcting for heterogeneity and Cox's partial likelihood estimates. We find that parameter estimates are very sensitive to the estimation procedure. We use the goodness-of-fit test and Hausman type specification test to choose the appropriate estimates to draw inference about the old-age security hypothesis, replacement effect and son/sex preference hypothesis for the Malaysian families. We find strong evidence for old-age security hypothesis and replacement effect and weak evidence for the son preference hypothesis.

KeyWords: Duration model  
Nonparametric maximum likelihood and partial likelihood estimates  
old-age security hypothesis  
replacement effect  
sex-preference hypothesis

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1. INTRODUCTION

In recent years there has been a lot of interest in the economic determinants of fertility in developing countries. Three controversial hypotheses in this regard are *old age security hypothesis*, *son or sex preference hypothesis*, and *replacement effect*. In the absence of well developed capital markets and publicly provided social security programs, parents would depend on their children for old-age support. Sex preference could be the result of old age insecurity of parents or it may simply be rooted in social norms or in tastes of the parents. The replacement effect, i.e., the responsiveness of fertility to infant and child mortality, arises when parents have a desired family size in mind which could again be the outcome of parents' old-age insecurity, or their deriving utility from family size. The presence of these effects will have consequences on population growth rate, savings rate, and income distribution of the subsequent generations. (See Raut [1986], Ben-Porath and Welch [1976], Heer[1983]).

Much of the empirical investigation of these effects has been carried out on completed family size and found controversial estimates ( see Cain [1983] Swidler [1983] ). Fertility decisions are made sequentially by a couple over their life-cycle and thus are affected by the changes in socio economic variables over the life-cycle. The perception about the degree of old-age insecurity, preference for son/daughter, and the occurrence of an infant or child death may depend at any time upon the number of surviving children, earnings profile of husband and wife, and stock of assets and therefore will vary over the life-cycle of a couple. Fertility decisions will also interact with the labor supply and savings decisions over the life-cycle of a couple.

Empirical analyses that are based on completed fertility will not be able to capture these dynamic effects, and also will have cohort biases in the parameter estimates since the young women who have not completed their reproductive periods will be dropped out of the sample and thus the sample will represent only those old women who survived until the survey date, i.e. only the women who are not right censored. Sequential econometric techniques are most appropriate in this situation.

Empirical studies that have incorporated sequential nature of fertility decisions are Heckman and Willis [1976], Newman and McCulloch [1984], Olsen and Wolpin [1982], Wolpin [1985], and Hotz and Millar [1988], Ben-Porath and Welch [1976]. The first two papers considered the hazard rate approach to fertility choice. In this framework, parents choose a contraception method in order to realize a feasible desired probability of live-birth in subsequent periods. I will follow this approach here. Hotz and Millar studied the interaction between fertility and savings decisions over the discrete time life-cycle periods using longitudinal household data on Panel Survey of Income Dynamics for the US. Wolpin formulated a simple life-cycle fertility decisions mechanism as a discrete time dynamic programming problem and estimated the structural parameters, and Olsen and Wolpin used waiting time regression framework to study the replacement effect. Both studies used the same data set as the one used in this paper. Ben-Porath and Welch used a regression analysis to estimate the effect of number of sons on subsequent birth intervals for Bangladesh families. While there have been a few attempts to study the replacement and sex preference hypotheses in a duration framework, no one has used this framework to test the old-age security hypothesis directly. This framework has the advantages that it takes into account the sequential nature of fertility decisions, the stochastic nature of the reproductive process, right censoring and the effects of time varying

measured and unmeasured heterogeneity. I use a continuous time multi-spell duration framework to study these three hypotheses.

Duration models face several econometric problems. If the unobserved heterogeneity among women that arises due to the differences in their nature's gift of fecundibility and omitted regressors are controlled for parametrically, the maximum likelihood (ML) estimates of the regressors as well as duration dependence parameters (i.e., the parameters of the base-line hazard function) are generally sensitive to the particular parameterizations of the heterogeneity distribution (Heckman and Singer [1982]). Heckman and Singer proposed a maximum likelihood estimation procedure in which the unobserved heterogeneity could be controlled for nonparametrically (NPML). Trussel and Richard [1985] found that NPML estimates for the regression coefficients are very sensitive to the specification of the base-line hazard function; however, when unobserved heterogeneity is ignored they found that the ML estimates are not sensitive to the base-line hazard specifications. This latter finding may suggest one to go ahead with the ML estimates ignoring heterogeneity until robust procedures are developed to control for unobserved heterogeneity. But, how parsimonious is this finding? In our analysis, we find that even when the unobserved heterogeneity is ignored, the ML estimates are sensitive to the base-line hazard specification, although less than the NPML estimates. As a most appropriate remedy, I use Cox's partial likelihood (PL) estimation procedure which is robust and consistent under any specification of the base-line hazard within the class of proportional hazard models without heterogeneity.

The sensitivity of estimation procedures to base line hazard specifications suggests that to use the ML or NPML estimation procedure, a criterion for model selection, especially among the non-nested models is very important. Several suggestions appear in the duration literature. Heckman

and Walker [1987] use goodness-of-fit criterion. Sharma [1987] uses a local specification testing based on LR tests in which a model is locally nested in a parameter space of higher dimension. In our empirical exercise, we find that two specifications accepted by the goodness-of-fit criterion produce very different ML estimates.

What is needed is a specification testing procedure that directly involves the parameter estimates. Therefore, Hausman specification testing is more appropriate in the duration context. When unobserved heterogeneity is absent, the standard Hausman test applies because the ML estimate provides the efficient estimate under a particular specification of a proportional hazard model such as Weibull or Gompertz and PL estimate provides a robust consistent estimate within the class of proportional hazard models. However, we run into the well known problem of estimating the dispersion matrix for the difference of these two estimates. I used the asymptotically equivalent m-tests criterion suggested by Newey [1985] and White [1987] to circumvent this problem; however, it requires us to assume that there is no censoring. Theoretical research is much needed to modify the test to handle censoring and unobserved heterogeneity.

Section 2 considers the modeling of marriage and fertility decisions. Section 3 describes the NPML and PL estimation procedures. Section 4 reports on the data set, variables used, and the sensitivity of the empirical estimates to different specifications and estimation procedures. Section 5 carries out the goodness-of-fit test and specification test. Section 6 highlights our findings on old age security hypothesis, son preference hypothesis, and replacement effect for Malaysian families based on the estimates recommended by the analysis of section 5. Section 7 summarizes the results.

## 2. A MODEL OF MARRIAGE AND FERTILITY DECISIONS

Economic analysis of marriage and fertility behavior has mostly been carried out in the static household production framework that was pioneered by Becker [1960]. Namboodiri [1972], and Moffit [1984] pointed out that fertility decisions are sequential decisions and interact with the evolution of various socio-economic variables over the life-cycle such as husband and wife's education, earnings profile of the household, together with other social norms. Heckman and Willis [1975], Newman and McCulloch [1984], Hotz and Millar [1988], and Wolpin [1984] modeled fertility behavior in the life-cycle framework and estimated their models using data from different countries. Heckman and Willis treated the hazard or risk of birth as the choice variable and studied how they evolve over the life-cycle of a couple as a function of the cost of contraception, age, birth parity, the time profile of earnings, and the cost of children in a discrete time dynamic programming framework. Newman and McCulloch followed a similar approach in continuous time to study the waiting time distributions. Hotz and Millar studied the interaction between fertility decisions and female labor supply decisions over the life-cycle and how they are jointly affected by earnings profiles of the husband and wife. None of these models could solve the decision rule explicitly<sup>2</sup> and instead they parameterized the reduced form decisions rules. Wolpin, however, modeled fertility decisions as to have or not to have a child in each discrete time over the life-cycle. He assumed that the number of surviving children yields utilities to parents, and then he estimated all

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<sup>2</sup>It is very difficult to derive the decision rules analytically. However, Newman [1987] solved the problem for a simplified model.



the structural parameters of the dynamic model. This is computationally formidable. I follow the hazard rate approach of Heckman and Willis, and Newman and McCulloch.

I consider only the family formation decisions of the households, and all other decisions such as labor supply or savings are assumed to be exogenously given. The family formation decisions are timing of marriage and timing and spacing of births. No economic agent has full control over any of these timings, although they can have partial control over these by the choice of a mix of instruments. For instance, using such instruments as dowry, health and beauty care, efforts on scholastic performance, and establishing social connections a woman can partially control her timing of marriage. Similarly, using a mix of contraceptive methods such as complete abstinence, pills, abortions, breast feeding etc., a woman can partially control the timing of her giving births. Denote the set of these instruments by  $\mathcal{U}$ .

Each individual searches in the marriage market for a suitable partner who can fulfill his/her desired timing and spacing of family. Assume for simplicity of analysis that once married, the partners do not divorce and that there are no births outside marriage.<sup>3</sup> I will treat the wife of the household as the decision maker. Over her life cycle a woman will visit the following biological states: susceptible to pregnancy, pregnancy, pregnancy resulting in miscarriage or still birth, pregnancy resulting in a live-birth, sterility as a result of menopause or death of husband.<sup>4</sup> Our interest in this paper is, however, to analyze the waiting time distributions between the states from the

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<sup>3</sup>These are consistent with the empirical facts of most traditional societies.

<sup>4</sup>I assume that there is no sterilization, and no abortions. Indeed, in our data set there are very few such incidence.

set  $S = \{\text{marry, pregnancy leading to a live-birth, sterility}\}$ , while controlling for her visits to other states that affect these waiting times.

Let  $(W, \mathcal{B}, \mu)$  be the underlying probability space on which all the following random variables are defined.

$K(\omega) =$  the number of live births over the life-cycle of the woman

$T_0(\omega) =$  age at marriage

$T_1(\omega) =$  duration between marriage and the first live-birth

$T_2(\omega) =$  duration between the first live birth and the second live birth

$T_k(\omega) =$  duration between the last two live-births, where  $K(\omega) = k$

$T_{k+1}(\omega) =$  duration between the last birth and the beginning of sterility.

Let  $\mathcal{T}_L = (T_0, T_1, \dots, T_{k+1})$  be the random vector of waiting times.

Let the random variable  $C(\omega)$  denote the number of miscarriages and still births that she may have in her life-cycle, and let  $\mathcal{T}_M = (M_1, M_2, \dots, M_{C(\omega)})$  be a random vector, where  $M_i$  is the age at which she had her  $i$ -th miscarriage,  $i = 1, 2, \dots, C(\omega)$ .

Let us denote by

$\mathcal{T}_L(t) = (T_0, T_1, \dots, T_r)$ , with  $r \leq k+1$ ,  $\sum_{i=1}^r T_i \leq t$ , the history of all

completed pregnancy durations up to her age  $t$ ,

$\mathcal{T}_M(t) = (M_1, M_2, \dots, M_s)$ ,  $M_s \leq t$ , the history of miscarriages and still births up to her age  $t$ ,

$M(t) =$  no of miscarriages and still birth up to her age  $t$ .

I shall denote by  $\langle X(t) \rangle_{t_1}^{t_2} = \left\{ X(t) \mid t \in [t_1, t_2] \right\}$ . Let the random variable  $\eta_0$  on  $(W, \mathcal{B}, \mu)$  denote the woman's gifts from nature such as luck, level of intelligence, and beauty that determine her probability of finding a suitable partner in the marriage market. Let the random variable  $\eta_j$  denote her fecundibility given to her by nature when she is at risk of giving  $j$ -th birth,  $j = 1, 2, \dots, K(\omega)$ . Let  $\eta = (\eta_0, \eta_1, \dots, \eta_{K(\omega)})$ . Note that  $\eta$  may be known to the couple, or the couple might learn over time using Bayesian or

other learning mechanisms. However, I assume that it is unknown to us.

At any time  $t$ , let her information set  $\Omega(t)$  contain the history of her live births, miscarriages, etc. Given the history of family planning decisions  $\langle u(t) \in \mathbb{U} \rangle_0^t$ , and  $\Omega(t)$ , assume that a couple chooses the hazard of a birth or marriage (if not already married) as follows:

The hazard function for marriage:

$$\begin{aligned}
 h_0 \left( t_0 \mid \langle u_1(t) \rangle_0^{t_0}, \Omega(t_0), \eta_0 \right) dt \\
 &= P \left( T_0 \in (t_0, t_0+dt) \mid \langle u_1(t) \rangle_0^{t_0}, \eta_0, T_0 \geq t_0 \right) \\
 &= \lambda_0 \left( t_0, \eta_0 \right) \cdot g_0 \left( \langle u(t) \rangle_0^{t_0} \right)
 \end{aligned} \tag{2.1}$$

Note that  $h_0$  is assumed to depend on  $\langle u(t) \rangle_0^{t_0}$ .

Parity and cause specific hazard functions for births

After marriage and already having her  $j$ -th live birth, her next transition will be either to a live birth, the cause is denoted as  $C = 1$ , or to sterility, the cause is denoted as  $C = 0$ , for all  $j \geq 1$ . I assume that these are the only two causes for a transition.

Parity  $j-1$  and cause  $C = c$  specific hazard function for her  $j$ -th transition is given by

$$\begin{aligned}
 h_j^c \left( t_j \mid \langle u(t) \rangle_{t_{j-1}}^{t_j}, \Omega(t_j), \eta_j \right) dt \\
 &= P \left( t_j \in (t_j, t_j+dt), C = c \mid \langle u(t) \rangle_{t_{j-1}}^{t_j}, \Omega(t_j), \eta_j, T_j \geq t_j \right) \\
 &= \lambda_j^c \left( t_j, \eta_j \right) \cdot g_j^c(u(t_j)) \quad j \geq 1, \text{ and } c = 1, \text{ and } 0
 \end{aligned} \tag{2.2}$$

where  $g(\cdot)$  measures the effectiveness of a contraceptive mix. For instance,

if  $u(t) = \text{abortion}$ ,  $g_j^1(u(t)) = 0$ .

Children provide services as well as utility to their parents. Like many other developing countries, Malaysia does not have a public social security system. The employers in the organized sector contribute to their employees' old-age pension fund. While the parents working in the organized sector would generally have their own wealth or pension fund to support themselves in their old age, the unskilled and poor parents may not have such sources of old age support. So for such parents children provide old-age pension. These parents would then like to have more children and would space them closely in their early age. While children provide services and utility to parents, they also cost money and time. At any moment  $t$  and given  $\Omega(t)$ , the woman would like to choose  $u(t) \in \mathbb{U}$  such that her expected life time utility is maximized subject to the above hazard functions and her budget constraints. I would not formulate her decision problem formally since it is difficult to solve analytically.

I make the necessary assumptions about the specification of her decision rule  $u_t = \Psi(\Omega_t, \eta)$  and the composite function  $g \circ \Psi$  such that (2.1) and (2.2) have the following proportional hazard representations:

$$h_0^1 \left( T_0 \in (t_0, t_0 + dt) \mid \langle X^0(t) \rangle_0^{t_0}, \eta_0 \right) = \lambda_0^1(t_0) \exp \left( X^0(t_0)' \beta_0 + \theta_0^1 \eta_0 \right) \quad (2.3)$$

and for  $j = 1, 2, \dots, J$ , and  $c = 0$ , and 1,

$$h_j^c \left( t_j \mid \langle X^j(t) \rangle_{t_{j-1}}^{t_j}, \eta_j \right) = \lambda_j^c(t_j) \exp \left( X^j(t_j)' \beta_j^c + \theta_j^c \eta_j \right) \quad (2.4)$$

where  $X^0(t)$ , and  $X^j(t)$  are the vector of time-varying covariates that include variables from  $\Omega_t$ , and also information about all previous demographic transitions.

The above hazard rate specifications capture measured heterogeneity in the term  $\exp(X'\beta)$ , unmeasured heterogeneity in the term  $\exp(\eta)$ , and the duration dependence (i.e., the dependence of the probability of exit at any time  $t$  on the time spent in the birth interval) in the term  $\lambda_0(t)$ .  $\lambda_0(t)$  could also be interpreted as the biological or natural hazard rate of a woman with average level of fecundibility. The above proportional hazard specification presumes that measured and unmeasured heterogeneity shifts the natural hazard rate proportionally.

Our main interest is to estimate the parameters of the hazard functions (2.3) and (2.4). While duration dependence is important from demographer's point of view, we are interested more in the  $\beta$ 's. For, these  $\beta$ 's will cast light on old-age security motive for having children, and replacement effects, i.e. whether an infant or child death induces a couple to have the next birth earlier. A strong replacement effect for higher parities will also be an evidence for old-age security motive.

### 3. ESTIMATION PROCEDURES

Typically for each household we have data on:  $t_0, t_1, \dots, t_r, t_{r+1}, \delta_{r+1}$ , and  $\langle X(t) \rangle_0^\tau$ , where  $r$  is the number of children,  $t_0$  = the age at marriage,  $t_1$  = duration between marriage and first live-birth, ...,  $t_r$  = duration between  $r-1$  and the  $r$ -th, i.e., last live-birth, and  $t_{r+1}$  = duration of the last event, which may be either her being censored in which case  $\delta_{r+1} = 0$ , or her attaining menopause in which case  $\delta_{r+1} = 1$ ,  $\tau$  being her age on the survey date. The likelihood of such an observation can be constructed as follows: Denote the combined hazard function by

$$h_j(t_j | \cdot) \equiv \sum_{c=0}^1 h_j^c(t_j | \cdot)$$

The corresponding survival function is given by

$$S_j(t | \cdot) \equiv P(T_j \geq t | \cdot) = \exp\left[-\int_0^t h_j(u | \cdot) du\right].$$

= probability of no live-birth or sterility for a period  $t$  after her  $j$ -1st live-birth.

The subdensity function of the  $j$ -th transition due to cause  $c$  is

$f_j^c(t | \cdot)$  = Probability that there is no transition before  $t$  due to either cause and there is a transition within an infinitesimally small interval of time after  $t$  due to cause  $c$ .

Note that

$$f_j^c(t | \cdot) = S_j(t | \cdot) \cdot h_j^c(t | \cdot).$$

The probability of her getting married at age  $t_0$  is given by  $f_0^1(t_0 | \langle X(t) \rangle_0^t, \eta_0)$ . Similarly, given  $t_0$ , the probability of  $(t_1, C = 1)$ , i.e., the probability of her neither giving a live-birth nor becoming sterile for a period of time  $t_1$  after her marriage and then giving a live-birth in the period  $(t_1, t_1+dt)$  is given by  $f_1^1(t_0 | \langle X(t) \rangle_{t_0}^{t_1}, \eta_1)$ . The process continues until she reaches her final transition due to menopause or she is censored. The likelihood of such an observation is

$$\begin{aligned} & f(t_0, t_1, \dots, t_r, t_{r+1} | \delta_{r+1}, \langle X(t) \rangle_0^\tau, \eta) \\ &= \prod_{j=0}^r \left( S_j(t_j | \cdot) h_j^1(t_j | \cdot) \right) \cdot S_{r+1}(t_{r+1} | \cdot) \left( h_{r+1}^0(t_{r+1} | \cdot) \right)^{\delta_{r+1}} \\ &= \prod_{j=0}^r \exp\left[-\int_0^{t_j} h_j(u | X^j(\tau(j-1)+u), \eta_j) du\right] \cdot h_j^1(t_j | X^j(t_j), \eta_j) \\ &\times \exp\left[-\int_0^{t_{r+1}} h_{r+1}(u | X^{r+1}(\tau(r)+u), \eta_j) du\right] \left[ h_{r+1}^0(t_{r+1} | X^{r+1}(t_{r+1}), \eta_{r+1}) \right]^{\delta_{r+1}} \end{aligned}$$

$$\begin{aligned}
&= \prod_{j=0}^r \left[ h_j^1(t_j | X^j(t_j), \eta_j) \cdot \left[ h_{r+1}^0(t_{r+1} | X^{r+1}(t_{r+1}), \eta_{r+1}) \right]^{\delta_{r+1}} \right. \\
&\times \left. \prod_{c=0}^1 \left[ \exp \left[ - \int_0^{t_j} h_j^c(u | X^j(\tau(j-1)+u), \eta_j) \right] \exp \left[ - \int_0^{t_{r+1}} h_{r+1}^c(u | X^{r+1}(\tau(r)+u), \eta_j) \right] \right] \right] \quad \dots \quad (3.1)
\end{aligned}$$

where  $h_j^0(. | .) \equiv 0^5$ .

Since I am going to use Heckman's CTM package (see Yi[1988] for details of the package), I shall specialize the base-line hazard functions of (2.3) and (2.4) to the the Box-Cox family:

$$\lambda_j^c(t_j) = \exp \left\{ \lambda_{0j}^c + \frac{t_j^{\lambda_{1j}^c} - 1}{\lambda_{1j}^c} \cdot \gamma_{1j}^c + \frac{t_j^{\lambda_{2j}^c} - 1}{\lambda_{2j}^c} \cdot \gamma_{2j}^c \right\}, \quad \lambda_{1j}^c < \lambda_{2j}^c \quad (3.2)$$

The Box-Cox family includes many well known distributions. I use the following three:

- (1)  $\lambda_{1j}^c = 0, \gamma_{2j}^c = 0$  : Weibull base-line hazard distribution
- (2)  $\lambda_{1j}^c = 1, \gamma_{2j}^c = 0$  : Gompertz base-line hazard distribution
- (3)  $\lambda_{1j}^c = 1, \lambda_{2j}^c = 2$  : Quadratic base-line hazard distribution

Note that Gompertz distribution is nested in quadratic distribution.

Weibull and Gompertz distributions are widely used for duration analysis in statistics and econometrics. Note that both models presume rapidly growing or decaying natural hazard rates -- Weibull at the polynomial rate, and Gompertz at the exponential rate. A quadratic model can, however, generate an intermediate shape for the natural hazard rate. The use of quadratic hazard is also motivated by evidence for such a shape in other fertility studies

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<sup>5</sup>The implicit assumption here is that a woman does not know before marriage if she has already attained her sterility.

(see, for instance, Trussel and Richard [1985], Tuma and Michael [1986]).

Most applications of duration models use maximum likelihood estimation procedure ignoring the unobserved heterogeneity. Even in this simpler case, would  $\beta$  estimates be sensitive to specifications of  $\lambda_0(t)$ ? Trussel and Richard [1985] find insensitivity of the maximum likelihood estimates of  $\beta$  to the above base-line specifications. However, our empirical results show that  $\beta$  estimates are sensitive (see table 1). This motivates us to consider Cox's partial likelihood estimation procedure.

Cox's Partial likelihood Estimation (PLE) procedure:

Cox[1972] in a path breaking paper suggested a robust estimation procedure for regression coefficients which is not sensitive to the base-line hazard function. Suppose that we have data of the following type

$$\left\{ t_{ji}^c, \delta_{ji}, \langle X_{ji} \rangle_{t_{j-1i}^c}^{t_{ji}^c} : j = 0, 1, \dots, E; c = 0, 1; i = 1, 2, \dots, n_j \right\}$$

where  $\delta_{ij} = 1$  if  $i$ -th observation completed the the  $j$ -th event, and  $\delta_{ij} = 0$  if the observation is censored; censored observations will have superscript 0.

Define the risk set  $\mathcal{R}_j(t)$  at  $t \geq 0$  by

$$\begin{aligned} \mathcal{R}_j(t) &= \text{the set of women who after completing their } (j-1)\text{-st event have} \\ &\quad \text{waited for at least a period of time } t \text{ for the occurrence of the} \\ &\quad \text{next event which is either a live-birth not leading to infant} \\ &\quad \text{death or her sterility.} \\ &= \left\{ i : t_{ji}^c \geq t, c = 0, 1 \right\}. \end{aligned}$$

The Cox's partial likelihood method maximizes the product of the conditional probabilities that the  $i$ -th person actually exits due to cause  $c$  at  $t_{ji}^c$ , given that it could have been anybody from the risk set  $\mathcal{R}_j(t_{ji}^c)$  over all observed points  $t_{ji}^c$ , with  $\delta_{ji} = 1$ . Using a commonly advocated adjustment procedure for ties in duration times, the method maximizes the following:



$$\mathcal{L}(\beta) = \prod_{j=0}^E \prod_{c=0}^1 \prod_{i=1}^{n_e^c} \left[ \frac{\exp \left( S_{ji} (t_{ji}^c)' \beta_j^c \right)}{\left[ \sum_{l \in \mathcal{R}_j(t_{ji}^c)} \exp \left( X_{jl} (t_{ji}^c)' \beta_j^c \right) \right]^{d_{ji}}} \right] \quad (3.3)$$

where,  $S_{ji} = \sum X_{jq}$ , summation is taken over  $\{q: t_{jq}^c = t_{ji}^c\}$ , and  $d_{ji}$  is the number of such ties. where  $n_j^c$  = number of uncensored observations for the  $j$ -th transition. Note that maximizing  $\mathcal{L}$  with respect to all  $\beta_j^c$ ,  $c = 0, 1$  and  $j = 1, 2, \dots, C$  is equivalent to maximizing (3.3) with respect to  $\beta_j^c$  separately for each  $c$  and  $j$ . It has been shown (see Wong [1986] for an account) that partial likelihood estimates of  $\beta$  is asymptotically distributed as  $N(\beta, I(\beta)^{-1})$ , where  $I(\beta) = \frac{\partial^2 \log \mathcal{L}(\beta)}{\partial \beta \partial \beta'}$  and the score vector  $U(\beta) = \partial \log \mathcal{L}(\beta) / \partial \beta$ , is asymptotically normally distributed with mean 0 and covariance matrix  $I(\beta)$ .<sup>6</sup> (See Kalbfleisch and Prentice [1980, 71-76], or Lawless [1981, 354-359]).

BIAS AND INSTABILITY OF PARAMETER ESTIMATES DUE TO OMITTED VARIABLES OR UNCONTROLLED HETEROGENEITY

I have given only a structural interpretation for  $\eta$ , namely as heterogeneity among couples in biological endowments such as fecundability that is observed by the couple but not by the econometricians. Heterogeneity can also arise statistically as a result of omitted regressors. In such a case, the parameter estimates of the included variables would suffer from omitted variables bias unless they are orthogonal to the omitted variables (Keifer [1988, 672-673]). The second way in which heterogeneity can arise is when some of the included covariates have measurement errors. For instance, the degree of old-age security that is perceived by a couple at any time

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<sup>6</sup> I developed a GAUSS program on IBMPc that can compute the above partial likelihood estimates for time varying covariates.

could not be directly observed and is to be instrumented by other variables in our empirical analysis. That means some of our covariates will involve measurement errors. The  $\beta$ -estimates in such situations will be both biased and inconsistent. Controlling for unobserved heterogeneity can correct some of these biases and inconsistency (Chamberlain [1985]). In the next section I empirically examine how sensitive the regressor estimates are when some of the significant regressors are dropped.

#### Heckman-Singer procedure for Controlling Unobserved heterogeneity

To control for unobserved heterogeneity in maximizing the likelihood function (3.1), several problems arise. The unobserved heterogeneity parameter could be treated as an individual and parity specific fixed effect or as a parity specific random effect. The main advantage of treating  $\eta$  as fixed effect is that one can avoid making any specific distributional assumptions about it. However, due to lack of enough data on individuals, the parameter estimates under fixed effect assumption will suffer from the well-known Neyman-Scott inconsistency syndrome. Chamberlain [1985] suggested a method of maximizing likelihood conditional on a sufficient statistics for  $\eta$  for multiple spells duration models within the exponential family of hazard functions; however, the procedure assumes that each individual has multiple spells and the heterogeneity is constant across spells. (For a criticism of the method, see Heckman and Singer [1985, pp.101]).

The other approach for controlling heterogeneity assumes that  $\eta$  is random. Many of the random effect duration models in the literature specify a parametric mixing distribution  $\lambda$  for  $\eta$ , where the distribution of  $\eta$  is known except for a finite number of parameters  $\rho$ ; and then integrating out  $\eta$  in (3.1) (numerically when analytical integration is difficult to calculate) one gets the marginal empirical distribution of the multiple spell durations as

$$\begin{aligned}
& g\left(t_0, t_1, \dots, t_{r+1} \mid \delta_{r+1}, \langle X(t) \rangle_0^\tau\right) \\
& = \int f\left(t_0, t_1, \dots, t_{r+1} \mid \delta_{r+1}, \langle X(t) \rangle_0^\tau, \xi, \eta\right) d\lambda(\eta) \quad (3.4)
\end{aligned}$$

where  $\xi$  is a vector of parameters from the hazard functions; one then maximizes this marginal likelihood with respect to the parameters  $\xi$  and  $\rho$ . Heckman and Singer [1982] found that the above procedure is too restrictive due to the assumption of a specific distribution of  $\eta$  to which the parameter estimates will be very sensitive (see their [1982] Table 2 for empirical evidence)<sup>7</sup>. They suggested a nonparametric maximum likelihood estimation (NPMLE) procedure to control for unobserved heterogeneity; the procedure in our context is described as follows:

Assume that sterility is not observable, i.e., the cause  $C = 0$  is non-observable. So I can drop the superscript  $c$  now on. Suppose  $\eta_j = c_j \theta$ ,  $j \geq 0$ , where  $c_j$  is the spell specific factor loading, and  $\theta$  is a mixing random variable on  $(W, \mathcal{B}, \mu)$  with the probability measure  $\pi$ . Note that this specification allows the unobserved heterogeneity to be correlated across spells. (3.4) now becomes

$$\begin{aligned}
& g\left(t_0, t_1, \dots, t_{r+1} \mid \delta_{r+1}, \langle X(t) \rangle_0^\tau\right) \\
& = \int f\left(t_0, t_1, \dots, t_{r+1} \mid \delta_{r+1}, \langle X(t) \rangle_0^\tau, \xi, c_0 \theta, \dots, c_{r+1} \theta\right) d\pi(\theta) \\
& = \sum_{\ell=1}^L f\left(t_0, t_1, \dots, t_{r+1} \mid \delta_{r+1}, \langle X(t) \rangle_0^\tau, \xi, c_0 \theta_\ell, \dots, c_{r+1} \theta_\ell\right) p_\ell \quad (3.5)
\end{aligned}$$

where,  $L$  is the number of supports,  $p_\ell \geq 0$ ,  $\ell = 1, \dots, L$ , and  $p_1 + \dots + p_L = 1$ .

The NPML procedure starts with  $L = 1$ , and increases the number of supports

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<sup>7</sup>Kiefer [1988, pp.676] points out that this sensitivity might be due to their choice of Weibul model.

until the likelihood ceases to increase. The procedure estimates the parameters  $\xi$ , and  $p_\ell$ 's and  $\theta_\ell$ 's simultaneously.

Although the procedure is quite general, it runs into few problems. First, since the likelihood function is not globally concave, the iteration procedure may not always converge, and when convergence obtains, it may not provide a global maximum. Secondly, asymptotic distribution for the estimates could not be derived when the the number of supports are not fixed a priori.

Following Heckman and Walker[1987], I also specialize the above non-parametric maximum likelihood procedure to a mover-stayer specification for heterogeneity (NPMS) which is simpler and does not have the above problems. The mover-stayer specification assumes that  $\eta_j$  takes only two values--sterile or fertile-- a proportion  $\pi_j$  of women during the j-th spell are sterile, and estimates  $\pi_j$  non-parametrically. The procedure is simpler, requires maximization of the likelihoods for each event separately, and it converges faster. In the following section I report NPMS estimates for Gompertz, Quadratic and Weibull models in table 2 and NPML estimates for Gompertz and Weibull specifications in table 5.

#### 4. EMPIRICAL RESULTS

##### THE DATA SET

The 1976-77 Malaysian Family Life Survey data are used for the analysis. This data set contains the event history data on 1262 households drawn randomly from private households consisting of at least one married woman of age less than fifty. These households represent quite well all the socio-economic strata in the country, and the data set has passed many consistency checks (On data reliability, see Haaga [1981]). Malaysian population consists of three racial groups - Malay, Chinese, and Indians -

Malays make up about 50% of the total population. Since government policies, customs, cultures, and social norms vary across racial groups, and also to keep computations task to a reasonable level, I carry out my investigation only on Malays. I choose the following variables for my preliminary analysis.

We must note that because of the sampling scheme, the parameter estimates of the marriage event may have choice-based sampling biases.

### Choice of variables

I will have the following variables:

RURAL = 1 if the household is in the rural sector, and = 0 otherwise  
ED\_LEVEL = level of education of the mother in number of years divided by 10  
MISCRG = number of miscarriages up to the present time  
CHLDTH = number of children died after six months old  
INFNTDTH = number of children died before six months old  
MON\_SEP = effective number of months the couples were geographically separated divided by 10  
DOWRY = amount of dowry paid during marriage divided by 100  
AGE\_LBT = age in months at last effective live birth divided by 100  
OLDAGE = 1 if the couple expect old-age support from their children and = 0 otherwise.

RURAL, ED\_LEVEL, DOWRY, AGE\_LBT, and OLDAGE are non-time varying, and the rest are time varying. In the demographic literature, such observed heterogeneity as MISCRG and MON\_SEP are generally not controlled for. If the oldage security motive is dominant among the Malays, we would expect a positive  $\beta$  for OLDAGE. However, this variable is an attitude variable and recorded only during the survey period; so the variable can have severe measurement errors when projected to earlier years. Moreover, the parents who do not have many children may report dependence on children for old-age

support as a matter of grievances and thus bias the effect of old-age motive for other parents.

**EMPIRICAL FINDINGS:**

A Comparison of Partial Likelihood estimates with Maximum Likelihood estimates with and without unobserved heterogeneity controlled for.

For Weibull, Gompertz, and quadratic models table 1 shows the maximum likelihood estimates without controlling for unobserved heterogeneity (MLE) and table 2 shows the nonparametric maximum likelihood estimates with mover-stayer heterogeneity (MSMLE). Table 3 shows the partial likelihood estimates (PLE). Several interesting observations can be drawn from these estimates

- <1> Whenever significant, the Weibull estimates in table 1 have in general higher absolute values than the estimates for the other two specifications. The exceptions to this pattern are the variables INFNTDTH, and RURAL. As is clear from table 2, when unobserved heterogeneity is controlled for, we do not observe the above systematic pattern for the Weibull model. A comparison of estimates from tables 1 and 2 reveal that for each model controlling for heterogeneity using mover-stayer specification does not change the estimates for age at marriage and first few birth intervals; the estimates are more sensitive for higher order birth intervals.
- <2> Although I find that the maximum likelihood estimates without controlling for heterogeneity for all the three models often agree with respect to sign and significance, they are not too close to each other. However, they are closer to each other than the corresponding NPML estimates.
- <3> The common pattern shown by these estimates is that

(a) from tables 1 and 2 it is clear that the age at marriage is related positively with ED\_LEVEL, RURAL AND OLDAGE. DOWRY has no effect on the age at marriage. Note that except for the effect of ED\_LEVEL, all other effects are counter-intuitive. However, the partial likelihood estimates in table 3 show that the effect of ED\_LEVEL is significantly positive, the effects of RURAL and OLDAGE are insignificant, and the effect of DOWRY is significantly negative on the age at marriage. The partial likelihood estimates for the marriage event seem more reasonable than the maximum likelihood estimates. However, this might be due to the choice-based sampling problems noted earlier.

(b) The relationships between the birth intervals and regressors are as follows:

< $\alpha$ > tables 1 and 2 show that the duration between marriage and first live-child is significantly negatively related to ED\_LEVEL, MISCRG, CHLDTH, INFNTDTH and age at marriage and the other variables are not significant. The partial likelihood estimates are also similar to the maximum likelihood estimates, with the notable differences that the partial likelihood estimates for MON\_SEP and RURAL are significant, and that the partial likelihood estimates for MISCRG, CHLDTH, and INFNTDTH are generally higher than the maximum likelihood estimates.

< $\beta$ > The maximum likelihood estimate of the effect of RURAL variable from tables 1 and 2 is significantly positive for durations 1-2, 3-4, 5-6, and 6-7. This is again counter-intuitive. However, the partial likelihood estimates for these parameters are not significant.

< $\gamma$ > All estimation procedures show that ED\_LEVEL has significantly negative effects on durations 0-1, 1-2, 2-3; all these estimates have comparable magnitudes. However, tables 1 and 2 show that it has no significant effect on birth intervals after the third, while the partial

likelihood estimates are significant for the durations 5-6 and 6-7.

<δ> Tables 1 and 2 establish that having their significantly negative effects on the duration between marriage and first live-birth, i.e., duration 0-1, MISCRG has significantly positive effect on all, INFNTDTH has significantly positive effect in almost all subsequent birth intervals, CHLDTH retains its negative impact on subsequent birth intervals up to the fifth child, and thereafter has no effect and MONSEP seem to have no effect on any duration. The partial likelihood estimates from table 3 more or less depict the same picture, with the notable differences that MISCRG is significant only up to the fourth live-birth instead of all births in the other two tables; when significant INFNTDTH has negative effect up to the fourth birth interval and thereafter it has positive effect. This finding about replacement effect is Olsen's findings on the same data set. MON\_SEP has significantly negative impact on durations 0-1, 2-3, and 5-6.

<ε> All the three estimation procedures show that the effect of AGE\_LBT is significantly negative for the first birth, not significant on the second birth, and thereafter it has significantly positive effects. However a closer look at these tables will show that maximum likelihood estimates with unobserved heterogeneity not controlled for are higher than both the maximum likelihood estimates with unobserved heterogeneity controlled for and the partial likelihood estimates. Moreover, while the coefficient estimates of AGE\_LBT for all birth intervals are close to each other in each model when heterogeneity is not controlled for, they differ substantially especially for higher order birth intervals and even become insignificant when unobserved heterogeneity is controlled for.

<φ> The effect of OLDAGE is most anomalous. Table 1 shows that it is significantly negative only for duration 5-6, and for 3-4 the effect is



significantly positive and otherwise not significant. Partial likelihood estimates show significantly negative effects for durations 5-6 and 5-7, although the effect on 3-4 is still positive. The estimates from table 2 show that OLDAGE has no significant effect on any birth interval except for negative impact on duration 4-5 with the Weibull model, and duration 5-6 with the quadratic model.

#### Omitted Variable Bias

Table 4 shows the maximum likelihood estimates for two specifications of the Weibull model--one including all the regressors (the estimates are in the first column for each event), and the other one excluding MISCRG, CHLDTH, INFNDTH, and MON\_SEP (the estimates are in the second column for each event). These excluded variables are presumably distributed identically across individuals and independent of the included regressors. The empirical findings of this exercise seem to point out the importance of heterogeneity arising from omitted variables. For note that a significant effect in the full model may become insignificant in the omitted variables model; compare, for example, the effect of RURAL on durations 3-4, 5-6, and 6-7.

#### Sensitivity of Heckman-Singer NPML estimates to Base-line hazard specifications

Earlier I have compared the sensitivity of the NPMS estimates to different base-line hazard specifications. Now I report the estimates for NPML procedure. Only the Gompertz and Weibull models were estimated with covariates RURAL, ED\_LEVEL, AGE\_LBT, OLDAGE, and only for four events, 0-1,

1-2, 2-3, and 3-4<sup>8</sup>. The reason for dropping the time-varying covariates is that the partial likelihood estimates (PLEs) are consistent for models with non-time varying covariates and thus I can compare NPMLEs with PLEs. Table 6 presents the estimates for both models with and without heterogeneity controlled for. For comparison, I also present the maximum likelihood estimates for quadratic model without heterogeneity and the partial likelihood estimates in the same table.

The table shows that for these two models while some of the significant maximum likelihood estimates that are close to each other when heterogeneity is ignored, may cease to do so when heterogeneity is controlled for. For, note that out of the ML estimates RURAL in duration 1-2, ED\_LEVEL in 0-1, AGE in 0-1, 2-3, 3-4, and OLDAGE in duration 3-4 (with a wrong expected sign) that are comparable when heterogeneity is ignored, the NPML estimates for RURAL in 1-2, and AGE in 2-3 diverge. However the opposite phenomenon is observed for RURAL in 2-3, and OLDAGE in 3-4.

When compared with the PL estimates, I find that while for some parameters the NPML corrects an estimate toward PLE as for instance, RURAL in 2-3, OLDAGE 3-4 in Weibull model, for other parameters the NPML estimates deviate from both PLE and each other, (for instance, ED\_LEVEL in 1-2, 2-3, AGE in 1-2, 2-3, OLDAGE in 1-2).

As regards to duration dependence, we observe that after the heterogeneity is controlled for non-parametrically, the shape of the baseline hazard function changes significantly for each specification. For event 1-2, figure 1 shows the base-line hazard functions for Gompertz and figure 2 for Weibull. Dotted curves correspond to NPMLE and continuous curves to MLE.

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<sup>8</sup> We also tried to estimate the quadratic model. The convergence did not obtain easily. So we dropped it.

Summarizing the above empirical findings, it is clear that

- <1> when unobserved heterogeneity is ignored, the maximum likelihood estimates of the regression coefficients for all three models most frequently agree with each other and with the partial likelihood estimates;
- <2> the omitted variables can affect the magnitude as well as the precision of the estimates;
- <3> controlling for heterogeneity nonparametrically may change the signs and significance of parameter estimates of a model when estimated ignoring heterogeneity;
- <4> MSMLEs are less sensitive to the base-line hazard specification than NPMLEs, and MSMLEs are comparable to PLEs;
- <5> Inference about duration dependence depends on its parameterization regardless of which estimation procedure is followed and is thus least dependable.

Therefore it is clear that specification diagnostics are necessary to choose a model.

## 5. GOODNESS-OF-FIT TEST, AND SPECIFICATION TEST FOR MODEL SELECTION

### GOODNESS-OF-FIT-TESTING:

The issue of how to choose a model is perhaps most important in the duration context. The likelihood ratio (LR) test procedure for nested specifications, and goodness-of-fit test procedure for non-nested specifications have been advocated for in the econometric duration analysis (see Heckman and Walker [1987]). Note that while LR test aids to choose between two competing models, none of which could be the true data generating process, the goodness-of-fit test is an absolute test against any alternative. However, the goodness-of-fit test could be sensitive to the choice of cells

which are arbitrary, and it may accept two models with different  $\beta$  estimates, as we see in our empirical findings below.

Table 6 shows the results on goodness-of-fit tests for different models. It is clear that the quadratic model is accepted more frequently than the other two specifications, and general heterogeneity specifications are also more often accepted. A closer look at table 6 reveals that choices of cells and the truncation period can lead to opposite test results for the same specification and the same parameter estimates. Note that for the covariate group 1, the Weibull model with a general heterogeneity specification and the quadratic model without controlling for heterogeneity are both accepted by the goodness-of-fit criterion (indeed for both models the cell probabilities are pretty close to each other for all truncation periods and cell divisions). However, their beta estimates (table 5 WB\_NPML and QUAD columns) differ quite significantly, e.g., estimates of AGE for 1-2, 2-3, and of OLDAGE for 1-2.

#### SPECIFICATION TESTING:

It is clear from the above results that a goodness-of-fit test cannot say much about the  $\beta$  estimates. One needs to carry out specification testing that directly involves the  $\beta$  estimates. Hausman type test is more appropriate and the test directly applies to the proportional hazard models without unobserved heterogeneity.

Let  $d = \hat{\beta}_{ML} - \hat{\beta}_{PL}$ , where  $\hat{\beta}_{ML}$  is the maximum likelihood estimates under the null specification of the base-line hazard function, and  $\hat{\beta}_{PL}$  is the partial likelihood estimates. It is known, at least for non-time varying covariates, that the  $\hat{\beta}_{PL}$  are consistent when the base-line hazard function is completely unknown although under the null specification it might be less efficient than  $\hat{\beta}_{ML}$ . Note that if the model is correctly specified,  $CHISQ = d'V^{-1}d$  is distributed as  $\chi^2$  with  $k$  degrees of freedom, where  $k$  is the

dimension of  $d$ , and  $V = V_{ML} - V_{PL}$ , where  $V_{ML}$ , and  $V_{PL}$  are the asymptotic dispersion matrices of  $\hat{\beta}_{ML}$  and  $\hat{\beta}_{PL}$  respectively (Hausman [1978]). In our case,  $V$  turns out to be not positive definite. So with some statistical abuse, I use  $V = V_{PL}$  which could, however, be justified as if we are testing the null hypothesis that  $H_0: \beta = \hat{\beta}_{ML}$ . The CHISQ values then become 25.81, 14.33, and 15.97 respectively, in which case all three models are rejected at 5% level ( $\chi_{12}^2(.05) = 21.0$ ) and Gompertz and quadratic are accepted at 1% level ( $\chi_{12}^2(.01) = 26.2$ ). I also carry out this testing for the NPML estimates for the events 0-1, 1-2, and 2-3, for Gompertz and Weibull models. The CHISQ statistics are 17.84 for Gompertz and 108.04 for Weibull. So the Weibull model is again rejected.

To get around the problem of non positive definiteness of  $V$ , I also follow the Newey-Tauchen  $m$ -testing framework which has been shown to be asymptotically equivalent to the Hausman test (Newey [1985]). However, since partial likelihood is defined only for non-censored observations, the method does not apply directly if censoring is present. So in order to apply the  $m$ -test, I deleted the censored observations from our sample to compute the following scores. This may, however, bias our inference.

Following Newey [1985] and White [1987], let  $\hat{m}_i(\hat{\beta}_{ML})$  and  $\hat{\pi}_i(\hat{\beta}_{ML})$  be respectively the the vector of maximum likelihood score and the partial likelihood score at  $\beta = \hat{\beta}_{ML}$ , the m.l.e. The test statistic is  $nR^2 \sim \chi_k^2$ , where  $n$  = number of observations,  $R^2$  is the non centered R-square statistics from regression of 1 on  $[\hat{m}_i(\hat{\beta}_{ML}), \hat{\pi}_i(\hat{\beta}_{ML})]$ . I computed this statistic for Gompertz, Weibull, and quadratic model for each transition, 0-1, 1-2, and 2-3, separately and all three jointly, they were all larger than 200. So all three specifications are rejected. Such unbelievably high values would make us suspect the validity of this test or the sensitivity of this test to the restriction of non-censored observations only. Theoretical investigations are

much needed in this area.

**IN SUM,** Our goodness-of-fit test results reveal that the quadratic model without unobserved heterogeneity has performed as good as the Weibull and Gompertz models with nonparametric heterogeneity specifications and better than the Weibull and Gompertz models without unobserved heterogeneity. Our specification test accepts the quadratic and Gompertz (which is nested in quadratic) specifications. Nonparametric evidence from other studies suggests a quadratic shape for the natural hazard rate for fertility events. On the basis of these, therefore, I present the NPMS estimates for the quadratic or quadratic-like models and the partial likelihood estimates in the next section to draw inferences about old-age security hypothesis, replacement effect, and sex-preference hypothesis.

## **6. PARAMETER ESTIMATES: OLD-AGE SECURITY, SON PREFERENCE, AND REPLACEMENT EFFECTS**

### Replacement Effect:

Replacement effect measures the responsiveness of the fertility decisions to an infant or child death. In the literature there has been some dispute as to whether infant/child mortality is exogenous or it depends on number of children. More children in a family may cause a higher infant/child mortality as due to sharing limited resources (see Heer [1983] on the controversy). Our hazard rate approach provides the percentage increase in the probability of having a child when there is an infant/child death for each parity. So our estimates do not suffer from this problem. QUAD columns of table 2 provides MSML estimates for quadratic model and table 3 provides the partial likelihood estimates. Estimates for CHLDTH and INFNTDTH reveal a strong evidence for replacement effect. This should be contrasted with the Wolpin's [1984] finding for weak replacement effect on the same data set. However, his estimation procedure is completely different.

A closer look at the estimates will reveal that CHLDTH has stronger effect than the INFNTDTH. Both are highest during the first birth interval, and then they decline during the higher birth intervals. For instance, during the first birth interval if there is an infant death then that will increase the probability of having a child at any time after that period by 60%(QUAD) to 80%(PLE); if there is a child death during the second birth interval then probability of having a child at any time afterwards will be increased by 23%(QUAD) to 54%(PLE). The combined effect of INFNTDTH and CHLDTH are positive up to the fifth birth interval. This implies that there is a demand for large family and thus the possibility of old-age security motive among Malays for having children:

Old-age Security Effect:

In the absence of publicly provided social security program, whether parents have to depend on children will depend upon their expected level of old-age wealth and pension fund. The degree of oldage insecurity felt by parents at different stages of their life-cycle is an attitudinal variable and is best measured by direct responses of the individuals. However, our sample recorded the parents' response only at the survey date. Moreover, as pointed out earlier, this response may not be free from measurement errors, and endogeneity problem. To circumvent these problems, I estimated the following logit model restricting the sample to the survey date observations

$$\text{Prob(OLDAGE} = 1) = \exp(X\beta) / [1 + \exp(X\beta)]$$

then use the estimated model to predict the time varying probabilities for all time periods. This variable is denoted as POLDAGE. I took the regressors as wife's earnings, age, education, race, and number of surviving sons, and husband's earnings. A higher POLDAGE in our terminology means a higher expected dependency on children for old-age support.

For the model specifications with POLDAGE, I did not try to get NPML estimates using Heckman's CTM package nor PL estimates using my partial likelihood package as they are very time intensive. However, using SAS, I estimated the Weibull and Log-logistic models with the same set of regressors as in table 3 but replacing the OLDAGE variable by POLDAGE. I used data on all three races. The parameter estimates for only POLDAGE variable are shown in table 7. It is clear that for both models the POLDAGE estimates are significantly positive in all parities. For instance, out of any two women, if one of them has one percent higher expected dependency on children would increase the probability of having early marriage by 20%, first birth by 14%, second birth by 8% and so on. This reflects how expected old-age dependency on children affect the timing of marriage and timing and spacing of children.

Son Preference Hypothesis:

The empirical literature found controversial evidence on sex preference (see Ben-Porath and Welch on this). Only studies that relate the effect of number of son (N\_SON) on the subsequent children are the ordinary least square analyses of Ben-Porath and Welch using Bangladesh data, De Tray [1984] using Pakistan data and Leung [1987] using Chinese sample of the same Malaysian data as mine. All these are based on closed birth intervals and thus have sampling bias of throwing away the incomplete birth intervals. These studies do not control for important determinants such as infant death, and child death. They find the effect of N\_SON on subsequent birth intervals are weak but positive up to five children.

We apply hazard rate approach to the problem. We obtained the maximum likelihood estimates for Weibul and Log-Logistic models with the same regressors as in table 3 using data for all three races. The parameter estimates of N\_SON only are given in table 7. The estimates are significantly negative for first five children with the exception of 2-3. We



get a bit stronger support for the son preference hypothesis compared to the previous studies.

Effect of Age on fecundibility of women:

Our PL estimates in table 3 and MSML estimates for quadratic model in the QUAD column of table 2 show that the women who get married older get their first child earlier. During the second birth interval, age at the first live-birth does not matter. For the higher order birth intervals, the older women take longer period to conceive, and this age effect is stronger, the higher is the parity.

**IN SUM**, we find strong evidence for oldage security hypothesis, and son preference hypothesis, and strong replacement effect up to the fifth child in the fertility behaviors of Malay population. We also find that older women take longer time to conceive except for the first two births. The higher is the level of education of the mother, the higher is the probability of her having a child earlier and this is significant up to her fourth child.

## 7. CONCLUSIONS

The following are the summary of our findings:

- <1> Unlike the Trussel and Richard findings, the maximum likelihood estimates (MLE) of the regressors are sensitive to base-line hazard specifications even when unobserved heterogeneity is ignored. However, MLEs are less sensitive than maximum likelihood estimates with a mover-stayer mixing distribution for heterogeneity (MSMLE), and MSMLEs are less sensitive than maximum likelihood estimates with a nonparametric general mixing distribution for heterogeneity (NPMLE). Therefore, specification testing is essential for choosing a model that can provide reliable inference and policy analysis.
- <2> Goodness-of-fit test as a criterion for model selection examines only the

predictive power of a model, and can be insensitive to the parameter estimates. We find empirical evidence for this. Therefore, a Hausman type specification test that directly involves the parameter estimates is more appropriate.

- <3> Newey's m-test for specification diagnostics which is asymptotically equivalent to Hausman's specification test yields very large chisquare values for all specifications (larger than 200). However, when we used the dispersion matrix of the partial likelihood estimates in the Hausman test, Weibul model was rejected and Gompertz and quadratic model were accepted. Theoretical work is needed to modify the Hausman test to be applicable in duration context.
- <4> Results from Hausman type specification test and goodness-of-fit test reveal that a quadratic model ignoring heterogeneity performs as good as Weibul and Gompertz models with nonparametric mixing distributions for heterogeneity.

Therefore, to test our hypotheses we use the partial likelihood estimates, MSMLEs for a quadratic model, and MLEs for two Log-Logistic models. (A Log-Logistic model can generate a quadratic shape for base line hazard function. This specification was chosen so that we could use the standard SAS package to estimate the MLEs). The following are our policy conclusions:

- <5> While the expectations about the extent of oldage insecurity may change over the course of life-cycle, a couple anticipating higher degree of oldage insecurity gets married earlier, and space their children earlier and closer. This negative impact of oldage insecurity on the subsequent birth intervals is true for all parities although the strength of the effect gets weaker for higher parities. Thus a significant part of the Malaysian households have larger families resulted from pension motive.

<6> A significant number of households would like to have their next birth early if there is either a child death or an infant death in the family. This effect is generally found to be stronger for a child death than an infant death and the combined effect is significant up to the fifth child.

<7> The effect of number of sons on subsequent birth intervals is significantly negative up to the fifth child. Thus the Malaysian families exhibit son preference. However, the son preference effects are found to be weaker in terms of t-statistics than the old-age security effects or replacement effects.

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TABLE 1: Weibull Model, Gompertz, and Quadratic Models with Time-varying covariates and without controlling for heterogeneity

VARIABLES	Marrg		Marrg		0-1		0-1		1-2		1-2		2-3		2-3		
	WBL	COMP	QUAD	WBL	COMP	QUAD	WBL	COMP	QUAD	WBL	COMP	QUAD	WBL	COMP	QUAD	QUAD	
CONSTANT	-1.668 (-11.90)	-1.022 (-6.13)	-3.173 (-12.59)	0.485 (2.16)	0.287 (0.97)	0.174 (0.41)	0.640 (2.82)	0.497 (1.48)	1.089 (4.14)	2.053 (8.71)	1.453 (4.39)	1.328 (3.91)					
RURAL	-0.148 (-1.35)	-0.191 (-1.31)	-0.034 (-0.32)	0.108 (0.98)	0.106 (0.76)	0.150 (1.30)	-0.184 (2.04)	-0.182 (1.43)	-0.259 (2.68)	0.187 (2.12)	0.084 (0.66)	0.010 (0.08)					
ED LEVEL	-1.012 (7.01)	-0.796 (-4.45)	-1.10 (-7.33)	0.671 (4.59)	0.522 (2.81)	0.611 (3.90)	0.543 (4.04)	0.213 (1.15)	0.307 (2.17)	0.783 (4.78)	0.438 (1.97)	0.438 (2.44)					
MISCERG	--	--	--	0.552 (3.74)	0.482 (2.72)	0.481 (3.13)	-0.949 (6.53)	-0.744 (4.06)	-0.872 (5.30)	-0.819 (5.66)	-0.618 (3.52)	-0.541 (3.55)					
CHLDTH	--	--	--	--	--	--	0.510 (3.81)	0.377 (2.06)	0.340 (2.59)	0.397 (3.41)	0.303 (2.00)	0.293 (2.66)					
INFNTDTH	--	--	--	0.673 (3.13)	0.555 (1.89)	0.593 (2.51)	-0.341 (3.03)	-0.254 (1.46)	-0.401 (2.66)	0.098 (0.97)	0.054 (0.37)	-0.020 (0.18)					
MONSEP	--	--	--	0.023 (0.48)	0.338 (0.63)	0.020 (0.40)	0.068 (0.81)	0.057 (0.72)	0.040 (0.71)	0.108 (1.09)	0.079 (0.62)	0.108 (1.21)					
AGE	--	--	--	0.538 (6.69)	0.443 (3.91)	0.515 (5.58)	0.196 (2.26)	0.063 (0.48)	-0.004 (0.04)	-0.493 (5.42)	-0.407 (3.21)	-0.446 (4.07)					
OLDAGE2	-0.459 (-4.50)	-0.439 (-4.22)	-0.194 (-1.18)	-0.016 (0.06)	-0.018 (0.07)	-0.067 (0.31)	0.045 (0.20)	0.041 (0.15)	-0.051 (0.29)	0.078 (0.33)	0.073 (0.21)	0.009 (0.03)					
DOWRY	2.805 (0.50)	3.241 (0.58)	2.161 (0.44)														

(table 1 continues to the next page) absolute t-values are in brackets.

(Table 1 continues . . . .)

VARIABLES	3-4 WBL	3-4 GOMP	3-4 QUAD	4-5 WBL	4-5 GOMP	4-5 QUAD	5-6 WBL	5-6 GOMP	5-6 QUAD	6-7 WBL	6-7 GOMP	6-7 QUAD	7-8 WBL	7-8 GOMP	7-8 QUAD
CONSTANT	3.162 (7.67)	2.297 (4.48)	1.547 (2.96)	4.078 (7.13)	3.050 (4.41)	-0.998 (0.95)	4.253 (7.35)	3.801 (5.46)	1.361 (1.43)	4.949 (6.19)	4.108 (3.63)	-0.923 (0.53)	4.434 (5.26)	3.313 (3.15)	-1.068 (0.80)
RURAL	-0.219 (1.48)	-0.153 (0.83)	-0.242 (1.60)	0.179 (1.14)	0.046 (0.21)	-0.056 (0.31)	-0.344 (1.67)	-0.283 (1.17)	-0.310 (1.36)	-0.491 (2.11)	-0.353 (1.09)	-0.468 (1.84)	-0.280 (1.15)	-0.211 (0.68)	-0.263 (1.02)
ED_LEVEL	0.319 (1.16)	0.133 (0.40)	0.027 (0.10)	0.109 (0.33)	-0.015 (0.03)	-0.217 (0.65)	-0.722 (2.33)	-0.738 (2.07)	-0.555 (1.42)	-0.058 (0.13)	-0.190 (0.35)	0.238 (0.45)	0.304 (0.52)	0.168 (0.23)	0.095 (0.16)
MISERG	-0.730 (4.48)	-0.602 (3.27)	-0.690 (4.34)	-0.956 (6.86)	-0.796 (4.58)	-0.735 (3.78)	-1.027 (6.17)	-0.899 (4.96)	-0.831 (4.20)	-1.138 (4.61)	-0.996 (3.72)	-0.749 (3.03)	-1.646 (6.24)	-1.285 (4.18)	-1.029 (3.99)
CHLDTH	0.051 (0.46)	0.095 (0.67)	0.105 (0.89)	0.238 (1.92)	0.182 (1.09)	0.232 (1.77)	0.024 (0.26)	0.024 (0.22)	0.033 (0.32)	-0.203 (1.88)	-0.144 (1.13)	-0.098 (0.73)	-0.005 (0.03)	0.020 (0.12)	-0.043 (0.30)
INFNTDTH	0.105 (1.21)	0.113 (0.95)	0.115 (1.03)	-0.297 (3.18)	-0.135 (1.10)	-0.043 (0.40)	-0.318 (2.82)	-0.259 (2.01)	-0.230 (1.76)	-0.448 (3.74)	-0.391 (2.77)	-0.369 (2.17)	-0.156 (0.99)	-0.078 (0.41)	-0.075 (0.48)
MONSEP	0.097 (1.19)	0.086 (0.78)	0.069 (0.88)	0.090 (0.97)	0.083 (0.81)	0.029 (0.28)	-0.018 (0.04)	0.000 (0.00)	-0.032 (0.08)	-0.154 (0.23)	-0.072 (0.10)	-0.235 (0.32)	-0.056 (0.08)	-0.042 (0.05)	-0.040 (0.03)
AGE	-0.597 (4.39)	-0.510 (3.04)	-0.535 (3.75)	-0.787 (4.71)	-0.642 (3.17)	-0.721 (4.45)	-0.448 (2.77)	-0.393 (2.15)	-0.417 (1.95)	-0.602 (3.21)	-0.535 (2.12)	-0.516 (2.07)	-0.641 (3.02)	-0.537 (2.05)	-0.705 (3.11)
OLDAGE2	-0.850 (4.44)	-0.564 (2.24)	-0.566 (2.23)	0.221 (1.16)	0.206 (0.77)	0.300 (1.21)	0.563 (1.69)	0.423 (1.12)	0.571 (1.48)	0.190 (0.45)	0.136 (0.26)	0.204 (0.43)	0.057 (0.10)	0.051 (0.07)	-0.195 (0.41)

Absolute t-values are in brackets.



TABLE 2 : MSM Estimates for Weibull, Gompertz, and quadratic Models (mover-stayer heterogeneity)

VARIABLES	Marrg		Marrg		0-1		0-1		1-2		1-2		2-3		2-3		
	WBL	COMP	QUAD	WBL	COMP	QUAD	WBL	COMP	QUAD	WBL	COMP	QUAD	WBL	COMP	QUAD	QUAD	
CONSTANT	-1.668 (-11.90)	-1.022 (-6.13)	-3.173 (-12.59)	0.485 (1.97)	0.287 (0.87)	0.175 (0.38)	2.266 (11.39)	1.889 (6.87)	1.985 (7.50)	2.518 (7.38)	2.219 (4.83)	2.01 (4.16)	2.518 (7.38)	2.219 (4.83)	2.01 (4.16)		
RURAL	-0.148 (-1.35)	-0.191 (-1.31)	-0.034 (-0.32)	0.108 (0.95)	0.106 (0.70)	0.150 (1.21)	-0.162 (-2.10)	-0.116 (1.12)	-0.292 (2.72)	-0.163 (-1.58)	-0.127 (-0.82)	-0.123 (-0.93)	-0.163 (-1.58)	-0.127 (-0.82)	-0.123 (-0.93)		
ED LEVEL	-1.012 (7.01)	-0.796 (-4.45)	-1.10 (-7.33)	0.671 (4.41)	0.522 (2.65)	0.611 (3.73)	0.610 (3.70)	0.383 (1.82)	0.356 (2.14)	0.760 (3.73)	0.521 (1.85)	0.584 (2.63)	0.760 (3.73)	0.521 (1.85)	0.584 (2.63)		
MISCRG	--	--	--	0.552 (3.66)	0.482 (-6.72)	0.481 (2.97)	-1.069 (6.72)	-0.944 (5.45)	-1.006 (5.65)	-0.500 (2.31)	-0.543 (2.46)	-0.597 (-3.29)	-0.500 (2.31)	-0.543 (2.46)	-0.597 (-3.29)		
CHLDTH	--	--	--	--	--	--	0.330 (2.56)	0.342 (1.91)	0.235 (1.81)	0.184 (1.61)	0.167 (1.04)	0.219 (1.68)	0.184 (1.61)	0.167 (1.04)	0.219 (1.68)		
INFNTDTH	--	--	--	0.673 (3.01)	0.555 (1.79)	0.593 (2.41)	-0.513 (-3.92)	-0.393 (-2.50)	-0.528 (3.34)	0.021 (0.15)	0.045 (0.24)	0.078 (-0.60)	0.021 (0.15)	0.045 (0.24)	0.078 (-0.60)		
MONSEP	--	--	--	0.023 (0.42)	0.034 (0.48)	0.020 (0.22)	0.004 (0.05)	0.018 (0.20)	0.004 (0.05)	0.090 (0.61)	0.073 (0.45)	0.103 (0.76)	0.090 (0.61)	0.073 (0.45)	0.103 (0.76)		
AGE	--	--	--	0.538 (5.60)	0.443 (3.52)	0.515 (4.95)	-0.054 (-0.60)	-0.060 (-0.50)	-0.020 (0.18)	-0.187 (-1.34)	-0.232 (-1.27)	-0.439 (-3.05)	-0.187 (-1.34)	-0.232 (-1.27)	-0.439 (-3.05)		
OLDAGE2	-0.459 (-4.50)	-0.439 (-4.22)	-0.194 (-1.18)	-0.016 (-0.05)	-0.018 (-0.04)	-0.067 (-0.22)	0.048 (0.22)	0.078 (0.25)	0.029 (0.10)	-0.049 (-0.15)	0.005 (0.01)	-0.123 (-0.39)	-0.049 (-0.15)	0.005 (0.01)	-0.123 (-0.39)		
DOWRY	2.805 (0.50)	3.241 (0.58)	2.161 (0.44)														

(table 2 continues to the next page) absolute t-values are in brackets.

(Table 2 continued ...)

VARIABLES	3-4		3-4		4-5		4-5		5-6		5-6		6-7		6-7		7-8		7-8			
	WBL	COMP	QUAD	WBL	COMP	QUAD	WBL	COMP	QUAD	WBL	COMP	QUAD	WBL	COMP	QUAD	WBL	COMP	QUAD	WBL	COMP	QUAD	
CONSTANT	3.891 (7.09)	2.791 (4.23)	1.622 (2.56)	4.724 (8.24)	3.552 (4.69)	-0.998 (-0.87)	3.884 (4.43)	3.996 (4.12)	1.362 (1.37)	3.946 (4.03)	4.155 (3.62)	-0.923 (-0.51)	5.310 (3.88)	4.979 (3.24)	-0.923 (-0.51)	3.946 (4.03)	4.155 (3.62)	-0.923 (-0.51)	5.310 (3.88)	4.979 (3.24)	-0.923 (-0.51)	
RURAL	-0.425 (-2.00)	-0.224 (-0.97)	-0.253 (-1.55)	-0.134 (-0.58)	-0.138 (-0.43)	-0.056 (-0.29)	-0.404 (-1.58)	-0.398 (-1.36)	-0.310 (-1.29)	-0.340 (-1.25)	-0.222 (-0.70)	-0.468 (-1.77)	-0.326 (-0.87)	-0.278 (-0.59)	-0.468 (-1.77)	-0.340 (-1.25)	-0.222 (-0.70)	-0.468 (-1.77)	-0.326 (-0.87)	-0.278 (-0.59)	-0.278 (-0.59)	
ED_LEVEL	0.168 (0.51)	0.070 (0.18)	0.026 (0.09)	-0.328 (0.98)	-0.164 (-0.35)	-0.217 (-0.63)	-0.142 (-0.29)	-0.200 (-0.37)	-0.555 (-1.37)	0.123 (0.16)	-0.126 (-0.15)	0.238 (0.43)	-0.282 (-0.29)	-0.184 (-0.15)	0.238 (0.43)	0.123 (0.16)	-0.126 (-0.15)	0.238 (0.43)	-0.282 (-0.29)	-0.184 (-0.15)	-0.184 (-0.15)	
MISCRCG	-0.618 (3.17)	-0.634 (2.99)	-0.698 (3.94)	-0.807 (4.41)	-0.852 (4.58)	-0.735 (3.69)	-1.109 (5.23)	-1.064 (4.92)	-0.831 (3.98)	-1.076 (4.39)	-1.143 (5.01)	-0.749 (2.96)	-1.072 (4.14)	-1.085 (4.14)	-0.749 (2.96)	-1.076 (4.39)	-1.143 (5.01)	-0.749 (2.96)	-1.072 (4.14)	-1.085 (4.14)	-1.085 (4.14)	
CHLDTH	0.013 (0.09)	0.078 (0.41)	0.103 (0.81)	0.164 (1.07)	0.110 (0.54)	0.232 (1.72)	-0.014 (-0.14)	-0.015 (-0.13)	0.033 (0.30)	-0.294 (-1.83)	-0.279 (-1.45)	-0.098 (-0.70)	-0.419 (-1.86)	-0.258 (-1.02)	-0.279 (-1.45)	-0.294 (-1.83)	-0.279 (-1.45)	-0.098 (-0.70)	-0.419 (-1.86)	-0.258 (-1.02)	-0.258 (-1.02)	
INFTDTH	-0.051 (-0.35)	0.061 (0.31)	0.113 (0.91)	-0.457 (-3.72)	-0.299 (-1.83)	-0.043 (-0.36)	-0.245 (-1.99)	-0.247 (-1.86)	-0.230 (-1.70)	-0.106 (-0.48)	-0.073 (-0.30)	-0.369 (-2.10)	0.050 (0.21)	0.020 (0.07)	-0.369 (-2.10)	-0.106 (-0.48)	-0.073 (-0.30)	-0.369 (-2.10)	0.050 (0.21)	0.020 (0.07)	0.020 (0.07)	
MONSEP	0.027 (0.30)	0.061 (0.47)	0.066 (0.48)	0.098 (0.875)	0.092 (0.81)	0.029 (0.28)	-0.089 (-0.178)	-0.074 (-0.14)	-0.032 (-0.06)	-0.562 (-0.78)	-0.556 (-0.82)	-0.225 (-0.32)	-0.088 (-0.13)	-0.073 (-0.11)	-0.562 (-0.78)	-0.556 (-0.82)	-0.225 (-0.32)	-0.088 (-0.13)	-0.073 (-0.11)	-0.073 (-0.11)	-0.073 (-0.11)	
AGE	-0.573 (-3.29)	-0.520 (-2.54)	-0.534 (-3.58)	-0.669 (-4.55)	-0.544 (-2.73)	-0.721 (-4.17)	-0.171 (-0.62)	-0.066 (-0.21)	-0.417 (-1.80)	-0.093 (-0.37)	-0.026 (-0.08)	-0.516 (-1.99)	-0.438 (-1.23)	-0.353 (-0.86)	-0.093 (-0.37)	-0.026 (-0.08)	-0.516 (-1.99)	-0.438 (-1.23)	-0.438 (-1.23)	-0.353 (-0.86)	-0.353 (-0.86)	-0.353 (-0.86)
OLDAGE2	-0.136 (-0.28)	-0.222 (-0.55)	-0.559 (-2.06)	0.623 (1.73)	0.438 (0.88)	0.300 (0.83)	0.504 (1.15)	0.444 (0.83)	0.571 (1.36)	0.237 (0.33)	0.250 (0.27)	0.204 (0.41)	-0.425 (-0.74)	-0.267 (-0.39)	0.237 (0.33)	0.250 (0.27)	0.204 (0.41)	-0.425 (-0.74)	-0.267 (-0.39)	-0.267 (-0.39)	-0.267 (-0.39)	

Absolute t-values are in brackets.

TABLE 3: Cox's Partial Likelihood Estimates (PLE)

VARIABLES	Marrg	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8
CONSTANT	.5349 (4.84)	-1.597 (6.48)	-.047 (0.17)	1.290 (4.15)	1.23 (3.36)	1.401 (3.21)	1.412 (2.49)	1.546 (2.48)	1.458 (1.66)
RURAL	-.060 (.57)	.154 (1.39)	-.074 (.60)	.054 (.45)	-.057 (.41)	.048 (.31)	-.058 (.32)	-.237 (1.20)	-.369 (1.48)
ED_LEVEL	-1.034 (8.15)	.521 (3.55)	.367 (2.44)	.654 (3.73)	.203 (.93)	.220 (.82)	-.511 (1.55)	.671 (1.79)	.114 (.22)
MISCRG		.609 (3.52)	.349 (4.95)	.270 (3.88)	.305 (3.96)	-.041 (.46)	-.018 (.17)	-.020 (.23)	-.201 (1.54)
CHLDTH		--	.538 (3.46)	.267 (2.28)	.134 (1.22)	.222 (2.04)	-.006 (.05)	-.057 (.57)	.024 (.18)
INFNTDTH		.832 (4.52)	-.149 (1.14)	.047 (.47)	.169 (1.68)	-.003 (.02)	-.267 (2.26)	-.389 (2.99)	-.063 (.36)
MONSEP		.097 (1.69)	.013 (.02)	.112 (2.04)	.053 (.82)	.031 (.39)	.382 (3.69)	-.195 (1.22)	-.076 (.54)
AGE		.591 (5.40)	.047 (.43)	-.423 (3.65)	-.470 (3.87)	-.475 (3.59)	-.345 (2.04)	-.319 (1.84)	-.267 (1.16)
OLDAGE2	-.209 (1.04)	-.055 (.28)	.253 (1.16)	.002 (.010)	-.521 (2.14)	.224 (.82)	.444 (1.49)	.444 (1.57)	.023 (.06)
DOMRY	.1054 (5.86)								

Absolute t-values are in brackets.

TABLE 4: Maximum likelihood estimates for Weibull Model.

VARIABLES	0-1	0-1	1-2	1-2	2-3	2-3	3-4	3-4	3-4	4-5	4-5	4-5
CONSTANT	0.485 (2.16)	0.565 (2.58)	0.640 (2.82)	0.532 (2.50)	2.053 (8.71)	2.087 (9.02)	3.162 (7.67)	3.207 (8.72)	3.162 (7.67)	4.078 (7.13)	3.975 (7.80)	
RURAL	0.108 (0.98)	0.094 (0.86)	-0.184 (2.04)	-0.245 (2.82)	0.187 (2.12)	0.274 (3.27)	-0.219 (1.48)	-0.084 (0.70)	-0.219 (1.48)	0.179 (1.14)	0.010 (0.08)	
ED LEVEL	0.671 (4.59)	0.606 (4.08)	0.543 (4.04)	0.335 (2.44)	0.783 (4.78)	0.485 (3.15)	0.319 (1.16)	0.140 (0.58)	0.319 (1.16)	0.109 (0.33)	-0.244 (1.01)	
MISCRG	0.552 (3.74)	* (6.53)	-0.949 (6.53)	* (6.53)	-0.819 (5.66)	* (5.66)	-0.730 (4.48)	* (4.48)	-0.730 (4.48)	-0.956 (6.86)	* (6.86)	
CHLDYH	--	* (3.81)	0.510 (3.81)	* (3.81)	0.397 (3.41)	* (3.41)	0.051 (0.46)	* (0.46)	0.051 (0.46)	0.238 (1.92)	* (1.92)	
INFNTDTH	0.673 (3.13)	* (3.03)	-0.341 (3.03)	* (3.03)	0.098 (0.97)	* (0.97)	0.105 (1.21)	* (1.21)	0.105 (1.21)	-0.297 (3.18)	* (3.18)	
MONSEP	0.023 (0.48)	* (0.81)	0.068 (0.81)	* (0.81)	0.108 (1.09)	* (1.09)	0.097 (1.19)	* (1.19)	0.097 (1.19)	0.090 (0.97)	* (0.97)	
AGE	0.538 (6.69)	0.543 (6.49)	0.196 (2.26)	0.180 (2.06)	-0.493 (5.42)	-0.556 (6.05)	-0.597 (4.39)	-0.684 (6.06)	-0.597 (4.39)	-0.787 (4.71)	-0.835 (5.64)	
OLDAGE2	-0.016 (0.06)	-0.116 (0.37)	0.045 (0.20)	0.211 (0.99)	0.078 (0.33)	0.262 (0.93)	-0.850 (4.44)	-0.800 (4.55)	-0.850 (4.44)	0.221 (1.16)	0.272 (1.34)	

Table 4 continues to the next page. Absolute t-values are in brackets.

(Table 4 continue .....)

VARIABLES	5-6	5-6	6-7	6-7	7-8	7-8
CONSTANT	4.253 (7.35)	3.835 (6.54)	4.949 (6.19)	5.288 (6.61)	4.434 (5.26)	3.081 (5.23)
RURAL	-0.344 (1.67)	-0.071 (0.44)	-0.491 (2.11)	0.092 (0.51)	-0.280 (1.15)	-0.350 (1.42)
ED_LEVEL	-0.722 (2.33)	-1.010 (3.29)	-0.058 (0.13)	-0.119 (0.29)	0.304 (0.52)	-0.149 (0.33)
MISCRG	-1.027 (6.17)	*	-1.138 (4.61)	*	-1.646 (6.24)	*
CHLDTH	0.024 (0.26)	*	-0.203 (1.88)	*	-0.005 (0.03)	*
INFNTDTH	-0.318 (2.82)	*	-0.448 (3.74)	*	-0.156 (0.99)	*
MONSEP	-0.018 (0.04)	*	-0.154 (0.23)	*	-0.056 (0.08)	*
AGE	-0.448 (2.77)	-0.585 (3.44)	-0.602 (3.21)	-1.116 (5.57)	-0.641 (3.02)	-0.530 (3.43)
OLDAGE2	0.563 (1.69)	0.403 (1.72)	0.190 (0.45)	0.510 (0.90)	0.057 (0.10)	0.312 (0.51)

Absolute t-values are in brackets.  
 Note: The second columns correspond to omitted variables Weibul Model.

TABLE 5: Comparing Parameter Estimates of Weibul and Gompertz models with and without controlling Heterogeneity those with PLE

VARIABLES	0-1						1-2					
	WB_ML	WB_NPML	GM_ML	GM_NPML	QUAD	PLE	WB_ML	WB_NPML	GM_ML	GM_NPML	WB_ML	PLE
CONSTANT	0.565 (2.58)	-0.430 (1.99)	0.310 (1.10)	-9.253 (8.98)	0.201 (0.47)	-1.453 (6.02)	0.532 (2.50)	-3.189 (1.99)	0.412 (1.38)	-0.872 (0.90)	0.809 (2.94)	-0.191 (0.68)
RURAL	0.094 (0.86)	0.082 (0.74)	0.097 (0.72)	0.009 (0.06)	0.141 (1.27)	0.145 (1.34)	-0.245 (2.82)	-0.434 (3.43)	0.218 (1.78)	-0.227 (1.73)	-0.285 (2.78)	-0.138 (1.12)
ED_LEVEL	0.606 (4.08)	0.594 (4.24)	0.459 (2.52)	0.472 (2.59)	0.553 (3.59)	0.470 (3.29)	0.335 (2.44)	0.142 (0.82)	0.061 (0.34)	-0.011 (0.06)	0.145 (1.05)	0.411 (2.77)
AGE	0.543 (6.49)	0.665 (9.08)	0.463 (4.39)	0.811 (5.30)	0.525 (6.01)	0.632 (5.94)	0.180 (2.06)	0.519 (4.32)	0.048 (0.40)	0.190 (1.14)	0.004 (0.04)	0.081 (0.73)
OLDAGE2	-0.116 (0.37)	-0.062 (0.23)	-0.109 (0.42)	-0.284 (1.12)	-0.157 (0.73)	-0.169 (0.85)	0.211 (0.99)	0.386 (1.64)	0.161 (0.63)	0.180 (0.63)	0.154 (0.85)	0.146 (0.70)
GAMA1	0.241 (7.48)	0.343 (7.82)	0.008 (0.08)	6.342 (11.85)	1.954 (6.81)	--	0.156 (3.04)	0.920 (10.69)	-0.467 (4.12)	-0.452 (3.92)	4.865 (11.81)	--
GAMA2					-3.236 (3.64)	--					-8.265 (15.19)	--

Table 5 continues in the next page. Absolute t-values are in brackets.

(Table 5 continues .....)

VARIABLES	2-3				3-4							
	WB_ML	WB_NPML	GM_ML	GM_NPML	WB_ML	WB_NPML	GM_ML	GM_NPML	QUAD	PLE		
CONSTANT	2.087 (9.02)	-18.060 (5.73)	1.535 (4.86)	0.916 (0.92)	1.408 (4.21)	1.290 (4.26)	3.207 (8.72)	-42.284 (4.84)	2.376 (5.20)	3.323 (1.90)	1.508 (3.41)	1.178 (3.36)
RURAL	0.274 (3.27)	-0.145 (1.09)	0.114 (0.91)	0.118 (0.93)	0.024 (0.22)	0.019 (0.16)	-0.084 (0.70)	-0.322 (1.69)	-0.723 (0.47)	-0.066 (0.41)	-0.148 (1.09)	-0.147 (1.08)
ED_LEVEL	0.485 (3.15)	0.221 (1.06)	0.215 (1.02)	0.198 (0.91)	0.273 (1.53)	0.591 (3.46)	0.140 (0.58)	-0.392 (1.35)	-0.048 (0.16)	-0.033 (0.11)	-0.172 (0.70)	0.187 (0.90)
AGE	-0.556 (6.05)	0.016 (0.09)	-0.473 (3.84)	-0.429 (3.20)	-0.515 (4.75)	-0.324 (2.92)	-0.684 (6.06)	-0.506 (2.63)	-0.579 (4.04)	-0.633 (3.83)	-0.616 (5.10)	-0.355 (3.06)
OLDAGE2	0.262 (0.93)	-0.062 (0.26)	0.177 (0.50)	0.182 (0.49)	0.105 (0.43)	0.034 (0.15)	-0.800 (4.55)	-0.556 (1.61)	-0.527 (2.30)	-0.533 (2.25)	-0.528 (2.19)	-0.551 (2.27)
GAMA1	0.231 (3.86)	1.354 (11.01)	-0.343 (2.47)	-0.335 (2.32)	6.636 (13.65)	--	0.331 (4.93)	1.648 (9.99)	-0.115 (0.79)	-0.101 (0.68)	8.492 (11.73)	--
GAMA2					-12.242 (19.06)	--					-16.607 (14.02)	--

Note: ML : Maximum likelihood estimates without controlling for heterogeneity  
 NPML : Maximum likelihood estimates with heterogeneity controlled for nonparametrically  
 PLE : Cox's partial likelihood estimates  
 WB : Weibul model  
 GM : Gompertz Model  
 Absolute t-values are in brackets.

TABLE 6: Predicted cell probabilities for different models and goodness-of-fit test results

No. of births in different cut-off points	Gompertz				Weibull				Quadratic				Actual cell probabilities
	COV1 ML	COV1 NPM	COV2 ML	COV2 MSML	COV1 ML	COV1 NPM	COV2 ML	COV2 MSML	COV1 ML	COV1 NPM	COV2 ML	COV2 MSML	
<b>4yrs. aft. mar.</b>													
n = 0	.158	.154	.144	.145	.145	.138	.130	.130	.135	.135	.122	.124	.090
n = 1	.460	.482	.473	.479	.546	.530	.555	.547	.530	.530	.531	.532	.536
n = 2	.243	.276	.243	.252	.233	.282	.236	.261	.263	.263	.268	.267	.321
n = 3+	.138	.088	.140	.124	.076	.049	.080	.061	.073	.073	.079	.077	.054
joint F-stat	70.74	32.64	63.40	51.94	38.65	16.13	32.50	18.09	20.99	20.99	16.71	16.69	--
<b>6yrs. aft. mar.</b>													
n = 0	.085	.081	.075	.075	.071	.069	.061	.062	.066	.066	.058	.059	.045
n = 1	.372	.346	.385	.370	.428	.343	.431	.389	.361	.361	.363	.362	.357
n = 2	.277	.320	.273	.295	.312	.367	.312	.350	.356	.356	.350	.352	.371
n = 3+	.266	.253	.267	.260	.189	.220	.196	.199	.216	.216	.227	.227	.227
joint F-stat	37.16	18.67	34.85	23.54	29.54	8.32	24.08	10.30	7.10	7.10	3.75	3.83	--
<b>8yrs. aft. mar.</b>													
n = 0	.059	.058	.052	.051	.052	.053	.045	.045	.051	.051	.045	.045	.016
n = 1	.295	.244	.310	.282	.314	.226	.320	.278	.237	.237	.249	.251	.282
n = 2	.271	.283	.264	.282	.325	.309	.315	.321	.329	.329	.314	.313	.300
n = 3+	.375	.415	.374	.384	.309	.412	.320	.357	.383	.383	.392	.392	.402
joint F-stat	35.10	31.36	31.57	24.87	55.41	35.02	45.13	27.66	30.62	30.62	22.24	21.74	--
<b>11yrs. aft. mar.</b>													
n = 0	.049	.050	.043	.042	.048	.048	.041	.041	.047	.047	.041	.041	.007
n = 1	.223	.179	.238	.211	.207	.167	.225	.204	.165	.165	.183	.191	.236
n = 2	.240	.206	.232	.226	.291	.211	.268	.228	.229	.229	.214	.210	.198
n = 3+	.488	.565	.487	.521	.455	.574	.466	.528	.559	.559	.561	.558	.560
joint F-stat	53.16	43.99	46.42	36.15	80.15	54.06	63.28	40.75	51.72	51.72	38.34	35.58	--
<b>16yrs. aft. mar.</b>													
n = 0	.047	.048	.042	.041	.047	.047	.041	.041	.047	.047	.041	.041	.003
n = 1	.172	.158	.192	.177	.145	.150	.171	.180	.147	.147	.169	.178	.206
n = 2	.198	.158	.188	.160	.219	.159	.201	.152	.159	.159	.148	.146	.158
n = 3+	.583	.636	.579	.622	.588	.644	.587	.627	.648	.648	.641	.635	.633
joint F-stat	56.06	47.67	47.59	37.80	70.89	57.51	52.95	44.53	52.13	52.13	41.93	40.89	--

Table 6 continues in the next page



(table 6 continues)

6yrs.aft.mar. joint F-stat	25.94	7.14	28.73	16.50	24.57	.36	22.37	7.60	1.80	1.50	1.29	--
8yrs.aft.mar. joint F-stat	9.42	.81	12.76	4.35	28.66	1.47	25.96	6.35	2.38	.59	.51	--
11yrs.aft.mar. joint F-stat	14.43	.88	16.36	4.76	33.89	4.48	28.04	3.87	7.32	2.34	.88	--
16yrs.aft.mar. joint F-stat	8.66	.10	10.66	.59	13.69	1.26	9.34	.84	2.38	.50	.88	--

These four rows correspond to the cells with  $n = 0$  and  $n = 1$  pooled together.

TABLE 7: Parameter Estimates for POLDAGE2 and N\_SON for Weibull and Log-logistic Models

VARIABLES	mrg	mrg	0-1	0-1	1-2	1-2	2-3	2-3	3-4	3-4
	WBL	LLOG	WBL	LLOG	WBL	LLOG	WBL	LLOG	WBL	LLOG
POLDAGE2	20.19 (42.1)	16.37 (37.5)	14.21 (30.6)	13.56 (33.7)	8.60 (30.3)	7.33 (23.6)	6.61 (25.1)	5.98 (26.3)	5.78 (24.5)	4.66 (23.2)
N_SON					-.017 (2.12)	-.001 (.04)	.002 (.30)	*	-.007 (1.30)	*

VARIABLES	4-5	4-5	5-6	5-6	6-7	6-7
	WBL	LLOG	WBL	LLOG	WBL	LLOG
POLDAGE2	3.10 (14.7)	3.37 (18.0)	3.41 (18.4)	2.94 (15.2)	2.93 (16.7)	2.46 (.91)
N_SON	-.011 (2.23)	*	.001 (.20)	*	.008 (1.88)	*

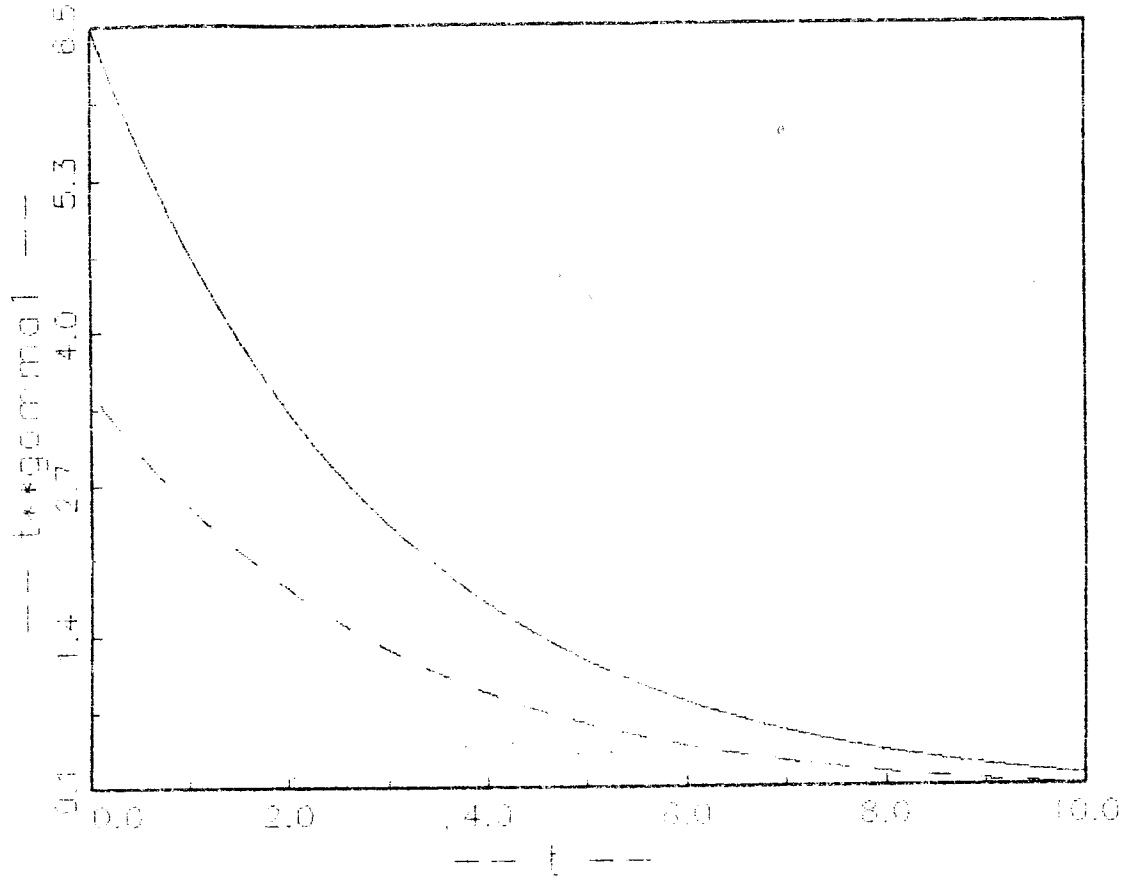
t-statistics in brackets

\* ==> convergence did not obtain in 50 iterations.

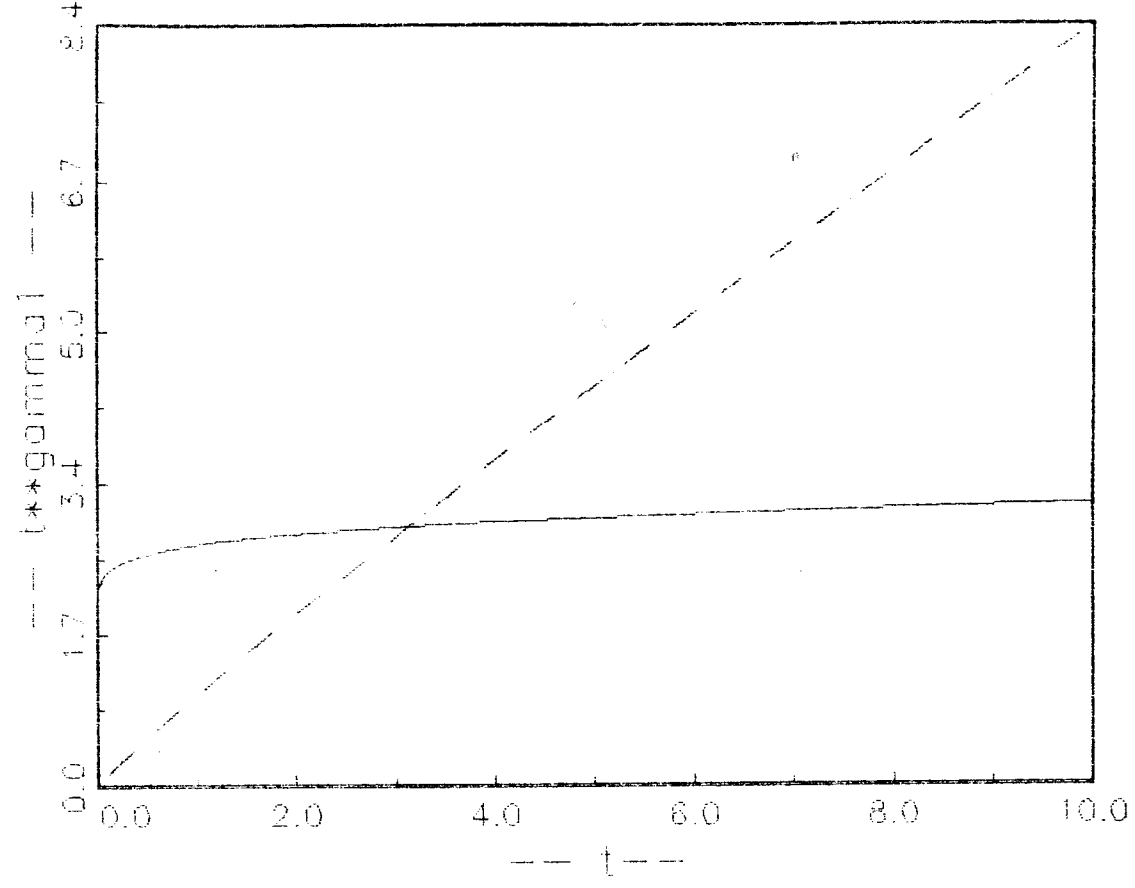
WBL ==> Weibull distribution

LLOG ==> Log-logistic distribution

# Gompertz hazard: 1-2 Table 5



# Weibul hazard: 1-2 table 5



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